Adaptive Division of Labor Control for Robot Group

Yusuke Ikemoto, Toru Miura, and Hajime Asama

Abstract— Division of Labor Control is advanced function for distributed autonomous robotic systems. Many studies focussing division of labor control inspired biological phenomenon have been reported. However, the optimality is not discussed because decentralized control is typically complicated. In this study, we propose the division of labor control method for robot group that enable adaptively select whether homogeneous state or heterogeneous state against working conditions and address the optimality by mathematical analysis. To evaluate the effectiveness of the proposed method, the computer simulations are carried out and we confirm that robot group implemented the proposed method inevitably organize the division of labor state with group performance improvement.

I. INTRODUCTION

Division of labor in a robot group system is an advanced collective behavior. Distributed autonomous robotic systems carries out given labor collectively: it is expected to yield group performance improvement with the division of labor control. In this study, adaptive division of labor control means the ability that enable robot group to automatically select whether homogeneous state or heterogeneous state according to working conditions by decentralized control.

Many studies using learning are related to such division of labor algorithms to realize a division of labor in a multi-agent system. A multi-agent reinforcement learning algorithm for cooperative behavior has been reported[1][2]. The division of labor achieved through learning algorithm or other computational optimization algorithm is expected to yield a collective performance improvement. However, in division of labor with learning algorithm, robots determine the better behavior after estimating the payoff of behaviors through experience, which means trial-and-error. Therefore, the use of learning algorithms is often accompanied by a time cost to learn it, along with complicated interaction protocols.

On the other hand, examining the biological world, divisions of labor model is reported[3][4][5]. Especially, social insects live with a group, called a colony, which organizes the division of labor, and which behaves adaptively against environmental conditions. Division of labor is one advanced social activity that is shown by many living beings. The common dynamics of the division of labor is that a heterogeneous state is generated by self-organization thorough interactions among individuals. The individual's internal non-learned property is exposed to the group state. The division of labor

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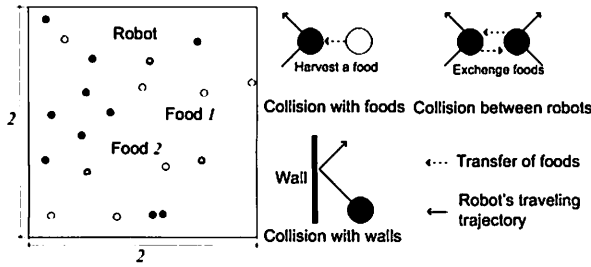
by biological-inspired model without trial-and-error process through a phase transition, it is however that its optimality is not guaranteed in their models.

The objective in this study is to propose division of labor control method for a robot group. In particular, the proposed method is evaluated using a situation of food forage labor by robot group. Then, it is proved theoretically that the group performance is certainly improved using proposed method, and the dynamical structure is elucidated to show how group performance is improved. Finally, the proposed algorithm and its dynamical behaviors are evaluated using computer simulations.

The contents of this paper are summarized as follows: Section 2 describes the assumed robot's task and working space, and defines the fitness to evaluate the group performance. In section 3, proposed division of labor control method is explained. Subsequently, it is proof that how the robot group implemented the proposed method achieves the adaptive division of labor against working conditions and works for group performance improvement by mathematical analysis. Section 5 describes evaluation of the adaptive division of labor with implementation of the proposed method to the problem provided in section 2. The salient results of this study are summarized in section 6.

II. ROBOT TASK

A. Task and Working space

1) Task overview: In this research, we used the following simple foraging problem, which includes a division of labor, through an interaction by the robot group on a working space, as shown in Fig. 1. The working space is defined as a 2×2 square. The number of robots is n in the working space. A robot moves in a working space, harvesting the food when a robot contacts with food. There are m kinds of food; the labor of robots is to assimilate the food that is harvested. The labor of robots is described as the following.

$$E_i = \{\text{Assimilating } i\text{-th food}\} \qquad (1)$$

The frequency of assimilation is described with strategy frequency $^j x_i$. For example, the j-th robot can assimilate the i-th food with frequency $^j x_i$ as the following.

$$^j x_i = \{j\text{-th frequency of } E_i\} \qquad (2)$$

$$\sum_i^m {}^j x_i = 1, \quad {}^j x_i \geqq 0$$

The fitness, that is working performance, is defined by the amount of assimilated food per unit time. Robots harvest and assimilate the m kinds of food in solitude if the group

Fig. 1. Working space and Collision rules of robots

Fig. 2. Division of labor for the robot group system. (a)Homogenous state. (b) Heterogeneous state

state is homogeneous. Robots harvest or exchange the food with other robots and assimilate a food using a specialized behavior if the group state is heterogeneous. Let ^jy_i be amounts of the i-th food that the j-th robot harvests per unit time.

2) *Robot behavior:* An robot has no directionality: it can move omnidirectionally. The initial states of both robots and food are random location. Robots have behaviors of three kinds for foraging: moving in a working space, harvesting food, and exchanging food with other robots. The labor of a robot E_i is assimilating the food as defined in the above subsections. For foraging, an robot generally moves with linear uniform motion with velocity $30_{[1/s]}$. The robot harvests the food on the working space if it contacts food. Interactions are carried out when the distance between a robot and another robot, food, or wall equals 0 (Fig. 1).

3) *Food-related behavior:* Food is replaced randomly in the working space. The food disappears from the working space if a robot harvests food. After that, new food is replaced randomly. Therefore, the food quantity is assumed to be constant. The working space has food of m kinds, where the number corresponds to the number of robot's strategies m. The quantities of i-th food are defined as C_i. For example, When $m = 2$, and the numbers of 1-th food and 2-th food are 7 and 3, respectively, we have $(C_1, C_2) = (7, 3)$. C_i do not correspond to ^jy_i. Here, C_i is the amount of food in the working space and ^jy_i is the amount of food that the j-th robot has for assimilating.

B. Labor performance definitions

The optimal ratio of the strategy frequency is defined as the same ratio existing in a food C_i. Therefore, the optimal strategy frequency is defined as $(^jx_1 : {}^jx_2 :, \cdots, : {}^jx_m) = (C_1 : C_2 :, \cdots, : C_m)$.

Let ^jG be the fitness matrix of j-th robot, which determines the amounts of food assimilated per unit time and the dynamics of strategy frequency ^jx. The fitness matrix is represented the matrix in which the i-th diagonal element is equal to ^jy_i and all of non-diagonal elements equal to $\sum_i^m {}^jy_i$ as follows.

$$^jG = \begin{bmatrix} ^jy_1 & & \sum_i^m {}^jy_i \\ & \ddots & \\ \sum_i^m {}^jy_i & & ^jy_m \end{bmatrix} \quad (3)$$
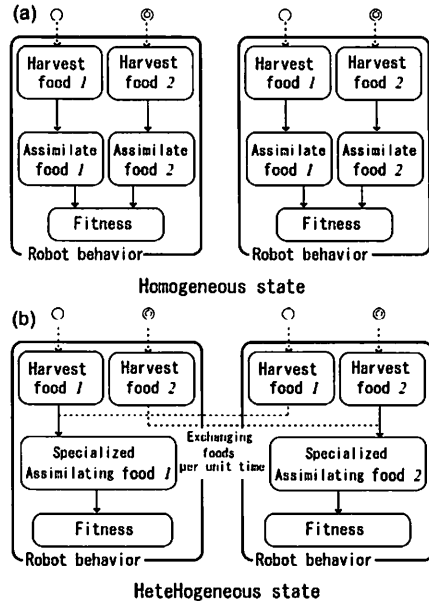
This fitness matrix is not explicitly given to an robot. A robot takes that through collecting foods. Then (3) depends on the contact frequency with foods. The ^jG consists of functions of ^jy_i because the amounts of food that a robot can assimilate are dependent on the amounts of food that a robot harvests. ^jG is determined according to the frequency with which a robot contacts with the food per unit time. For example, $^jg_{12}$, that is the element of ^jG, is the fitness when $^jx_1 = 1$ and $^jx_2 = 1$. The term of $^jg_{12}{}^jx_1{}^jx_2$ is one for the given fitness and corresponds to the amount of food that the robot can assimilate per unit time. The robot fitness $^j\phi$ is obtained using the summation of those terms.

In addition, the fitness given by j-th assimilation of food i is described as $^jf_i({}^jx)$. Without interactions among robots, i.e., no exchanges of food to be assimilated, the fitness simply corresponds to $\left({}^jG^jx\right)_i$, where $(\cdot)_i$ denotes the i-th row element, which is regarded as the expected value or degree of demand of each labor. In the case of robots' interaction with other robots, we define $^jf_i({}^jx)$ with interaction terms between other robots as follows.

$$^jf_i({}^jx) = \left({}^jG^jx\right)_i + {}^jh_i \quad (4)$$

Therein, ^jh_i denotes the fitness that is generated from interactions . In interactions, the summations of food to be assimilated in both robots must have a law of conservation of amount. Therefore, $\sum_j^n {}^jh_i = 0$ must be satisfied at any time. With these fitness, because robots carry out activities with strategy frequency ^jx, the j-th robot fitness is obtained as $^j\phi = \sum_i^m {}^jx_i{}^jf_i({}^jx)$. Thereby, we can determine the group mean fitness as

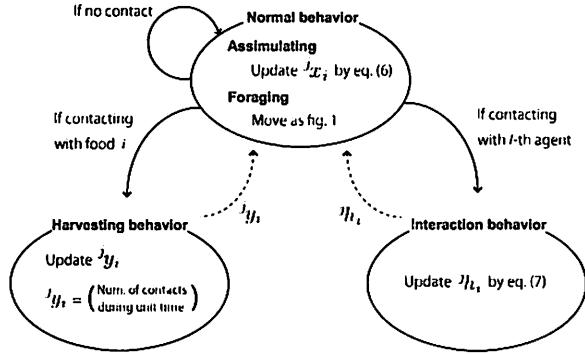$$\langle ^j\phi \rangle = \frac{1}{n}\sum_j^n {}^j\phi. \quad (5)$$

Fig. 3. Transition rules of robot's behavior.

III. DIVISION OF LABOR CONTROL

A. Rules of a behavior decision and an interaction

A simple gradient method can not be used, e.g. $\frac{d^j x_i}{dt} = \frac{\partial \langle^j \phi \rangle}{\partial^j x_i}$, because of the condition $\sum_i^m {}^j x_i = 1$. The optimization problem is transformed using a gradient projection system[6]. With the projection, the optimization on $\sum_i^m {}^j x_i = 1$ is described using the replicator equation, as the following,

$$\frac{d^j x_i}{dt} = {}^j x_i \left({}^j f_i ({}^j x) - {}^j \phi \right) \qquad (6)$$

The interaction term ${}^j h_i$ is determined with respect to exchanging labor demands. Let ${}^j p_i$ be food to be assimilated that a robot transports to another robot. To satisfy the condition of $\sum_j^n {}^j h_i = 0$, ${}^j h_i$ simply is described with mutual diffusion of ${}^j p_i$ as

$${}^j h_i = D \left({}^l p_i - {}^j p_i \right), \quad D = \begin{cases} \alpha, & \text{if interaction} \\ 0, & \text{else} \end{cases} \qquad (7)$$

Therein, $l \in N, \neq j$ denotes the number of robots that interact with the i-th robot; α represents the value at which robots can exchange their fitness. If the ${}^j p_i$ is the labor the robot must carry out, it is expected that labor and actual food to be assimilated can be exchanged among robots. With this consideration, the ${}^j p_i$ is determined as the following equation.

$${}^j p_i = \left({}^j G^j x \right)_i \qquad (8)$$

Therein, ${}^j p_i$ means not only the amount of food that an robot collects per unit time but also the expected fitness with ${}^j x_i$. The interaction defined by (8) generates the flows of expected fitness in a group. In addition, ${}^j h_i$ works as the global feedback to a robot's ${}^j x_i$ with differences of ${}^j p_i$ among robots.

B. Analysis for adaptability and optimality

1) Analysis setup: In this section, mathematical analyses of two kinds are performed on a two-dimensional labor situation(m = 2 and n = 10). In concrete terms, the proposed method constantly organizes the homogeneous or heterogeneous state with increasing group mean fitness $\langle^j \phi \rangle$.

Here, it is assumed that interactions among robots are carried out uniformly and continuously. Therefore, the time evolution of D is assumed to be continuous and a robot interacts in mean field approximation as

$${}^j h_i = D \left(\langle^j p_i \rangle - {}^j p_i \right), \qquad (9)$$

where $\langle \cdot \rangle$ is the mean related to j. Furthermore, ${}^j p_i$ is described by the function of ${}^j x_i$.

$${}^j p_1 = g_{12} - (g_{12} - g_{11}) {}^j x_1$$
$${}^j p_2 = g_{21} - (g_{21} - g_{22}) {}^j x_2$$

${}^j f_i \left({}^j x_1, {}^j x_2 \right)$ are also described by ${}^j x_i$; this is simply described as ${}^j f_i$.

$${}^j f_1 = (1 - D) {}^j p_1 + D \langle^j p_1 \rangle$$
$${}^j f_2 = (1 - D) {}^j p_2 + D \langle^j p_2 \rangle. \qquad (10)$$

(6) is transformed as the following.

$$\frac{d^j x_1}{dt} = {}^j x_1 (1 - {}^j x_1) \left({}^j f_1 - {}^j f_2 \right)$$
$$\frac{d^j x_2}{dt} = {}^j x_2 (1 - {}^j x_2) \left({}^j f_2 - {}^j f_1 \right), \qquad (11)$$

The dynamical system represented by (11)has three fixed points on S_2: $({}^j x_1, {}^j x_2) = (1, 0)$, $(0, 1)$, and $({}^j x_1^*, {}^j x_2^*)$. They satisfy ${}^j f_1^* = {}^j f_2^*$ and

$$x_1^* = \frac{g_{12} - g_{22} - Dk \langle x_1 \rangle}{(1 - D) k}$$
$$x_2^* = \frac{g_{21} - g_{11} - Dk \langle x_2 \rangle}{(1 - D) k}, \qquad (12)$$

where $k = g_{12} - g_{11} + g_{21} - g_{22}$.

To provide insight through analyses, the case of no interaction, which means $D = 0$, is analyzed first. When $D = 0$, (10) are transformed to ${}^j f_i = {}^j p_i$. In this case, the fixed point $({}^j x_1^*, {}^j x_2^*)$ is described specifically as (x_1', x_2'). With (12),

$$x_1' = \frac{g_{12} - g_{22}}{k}, \quad x_2' = \frac{g_{21} - g_{11}}{k}. \qquad (13)$$

Then, the strategy frequency converges to one of $({}^j x_1, {}^j x_2) = (1, 0)$, $(0, 1)$, and $({}^j x_1', {}^j x_2')$. Meanwhile, (6) is known to have a unique stable fixed point on S_2 if ${}^j f_i ({}^j x)$ is a monotonically decreasing function related to ${}^j x_i$[6]. Consequently, when the conditions $g_{12} - g_{22} > 0$ and $g_{21} - g_{11} > 0$ are satisfied, the fixed point $({}^j x_1', {}^j x_2')$ is stable at $D = 0$. In addition, the conditions by which the fixed point $({}^j x_1', {}^j x_2')$ is not included on borders of ${}^j x_1' + {}^j x_2' = 1$ and outside of the region are $g_{12} - g_{11} > 0$ and $g_{21} - g_{22} > 0$.

In the situation in which ${}^j G$ does not satisfy those conditions, the robots need not organize a heterogeneous state because replicator dynamics in a robot have the maximization principle of ${}^j \phi$. Moreover, the optimal state is that all of j-th strategy frequency converges to $({}^j x_1, {}^j x_2) = (1, 0)$ or $(0, 1)$, and homogeneous state always becomes optimal. Therefore, these situations are deselected from those cases addressed in this paper. Then, the robot mean fitness is described as

$${}^j f_1 = {}^j f_2 = {}^j \phi' = -\frac{|{}^j G|}{k} \text{ at any } j.$$

2) Condition of differentiation:

2) *Condition of differentiation:* For $D > 0$, $^j f_i(^j x)$ is not a monotonically decreasing function at any time because (6) includes the (9) term. Let ζ, ξ, and η be ratios of the number of robots located at $(1,0)$, $(0,1)$, and $(^j x_1^*, ^j x_2^*)$, respectively, and satisfy $0 \leq \zeta, \xi \eta \leq 1$ and $\zeta + \xi + \eta = 1$. Additionally, $\langle ^j x_i \rangle$ is determined using a self-consistent method.

$$
\begin{aligned}
\langle ^j x_1 \rangle &= \zeta \cdot 1 + \xi \cdot 0 + \eta \cdot ^j x_1^* \\
\langle ^j x_2 \rangle &= \zeta \cdot 0 + \xi \cdot 1 + \eta \cdot ^j x_2^*
\end{aligned}
\tag{14}
$$

With (13) and (14),

$$
(\langle x_1 \rangle, \langle x_2 \rangle) = \left(\frac{Q_1 + Q_4}{kQ_3}, \frac{Q_2 - Q_4}{kQ_3} \right)
\tag{15}
$$

$$
(x_1^*, x_2^*) = \left(\frac{Q_1}{kQ_3}, \frac{Q_2}{kQ_3} \right)
\tag{16}
$$

where $Q_1(\zeta, D) = g_{12} - g_{22} - \zeta kD$, $Q_2(\xi, D) = g_{21} - g_{11} - \xi kD$, $Q_3(\zeta, \xi, D) = 1 - \zeta D - \xi D$, and $Q_4(\zeta, \xi) = \zeta(g_{21} - g_{11}) - \xi(g_{12} - g_{22})$. These satisfy $Q_1 + Q_2 = kQ_3$ and $\zeta Q_2 - \xi Q_1 = kQ_4$.

In addition, $0 \leq \frac{Q_1 + Q_4}{kQ_3}, \frac{Q_2 - Q_4}{kQ_3} \leq 1$ and $0 \leq \frac{Q_1}{kQ_3}, \frac{Q_2}{kQ_3} \leq 1$ are satisfied with $0 \leq ^j x_i \leq 1$ and $0 \leq \langle ^j x_i \rangle \leq 1$.

We carry out stability analysis with particular emphasis on the time evolution of $^j x_1$. Let J_1, J_2, and J_3 be Jacobians around the neighborhood of fixed points $(1,0)$, $(0,1)$, and $(^j x_1^*, ^j x_2^*)$, as

$$
\begin{aligned}
J_1 &= \frac{(1-D)Q_2}{Q_3} \\
J_2 &= \frac{(1-D)Q_1}{Q_3} \\
J_3 &= -\frac{(1-D)Q_1 Q_2}{kQ_3^2}.
\end{aligned}
\tag{17}
$$

Therein, $\langle \cdot \rangle$ is $^j x_i^*$-independent because n is assumed to be large. It is assumed that all of the strategy frequency is on either of the fixed points, as described above.

Hereinafter, the regions that guarantee linear stability of D, ζ, ξ and η with (17) in situations of one-cluster states, two-cluster states, and three-cluster states. The cluster existence conditions are summarized in Table I. Linear stability conditions of each case 1–7 are obtained using Jacobians (17). Let Reg_1, Reg_2, and Reg_3 be the regions of D, ζ, ξ, and η that respectively satisfy the stability conditions of one-cluster states, two-cluster states, and three-cluster states. According to Table I, the existence conditions of stable cluster are as follows:

$$
\begin{aligned}
\text{Reg}_1 &= \{ (\zeta, \xi, \eta) = (0,0,1), D < 1 \} \\
\text{Reg}_2 &= \{ \zeta + \xi = 1, \eta = 0, Q_1 < 0, Q_2 < 0, D > 1 \} \\
\text{Reg}_3 &= \{ \emptyset \}.
\end{aligned}
$$

To summarize, at $1 - D < 0$ $(1 - D > 0)$, the division of labor is organized (disorganized). Therefore, the division is derived when degrees of an interaction term are dominant compared to a fitness term in (4).

TABLE I
CLUSTER EXISTENCE CONDITIONS

	case	ζ	ξ	η	J_1	J_2	J_3
one-cluster	case 1	1	0	0	negative	positive	positive
	case 2	0	1	0	positive	negative	positive
	case 3	0	0	1	positive	positive	negative
two-cluster	case 4	ζ	0	η	negative	positive	negative
	case 5	0	ξ	η	positive	negative	negative
	case 6	ζ	ξ	0	negative	negative	positive
three-cluster	case 7	ζ	ξ	η	negative	negative	negative

C. Group performance analysis

This subsection specifically addresses the increase and decrease of group mean fitness $\langle ^j \phi \rangle$. The necessary conditions for the increase of $\langle ^j \phi \rangle$ correspond to the stable regions of homogeneous or heterogeneous state provided in the previous subsection. Corresponding to the above, the proposed algorithm realize an adaptive division of labor.

The group mean fitness is determined as the following with ζ, ξ, η, D using a self-consistent analysis.

$$
\begin{aligned}
\langle ^j \phi \rangle &= \zeta(1 \cdot f_1(1) + 0 \cdot f_2(0)) \\
&+ \xi(0 \cdot f_1(0) + 1 \cdot f_2(1)) \\
&+ \eta(x_1^* \cdot f_1(x_1^*) + x_2^* \cdot f_2(x_2^*)).
\end{aligned}
\tag{18}
$$

Herewith,

$$
\begin{aligned}
(1-D)x_1^* + D\langle x_1 \rangle &= \frac{Q_1 + DQ_4}{PQ_3} = x_1' \\
(1-D)x_2^* + D\langle x_2 \rangle &= \frac{Q_2 - DQ_4}{PQ_3} = x_2',
\end{aligned}
$$

we have

$$
\begin{aligned}
x_1^* f_1^* + x_2^* f_2^* &= \frac{g_{12}g_{21} - g_{11}g_{22}}{k} = \phi' \\
f_1(1) - \frac{g_{12}g_{21} - g_{11}g_{22}}{k} &= -x_1' J_1 \\
f_2(1) - \frac{g_{12}g_{21} - g_{11}g_{22}}{k} &= -x_2' J_2
\end{aligned}
\tag{19}
$$

Therefore, substituting (19) to (18), we have the following as a simple relationship,

$$
\langle ^j \phi \rangle = \phi' - \zeta x_1' J_1 - \xi x_2' J_2
\tag{20}
$$

The first term of the right side in (20) is the value depending on $^j G$, not ζ, ξ, η, and D. When $J_1 > 0$ and $J_2 > 0$, in this case, $D < 1$ is satisfied, $\langle ^j \phi \rangle$ decreases with ζ or ξ increases. Therefore, $\langle ^j \phi \rangle$ become maximum with $\zeta = 0$ and $\xi = 0$. These conditions correspond to Reg_1. When $J_1 < 0$ and $J_2 < 0$, in this case $D > 1$ and $Q_1 < 0$, $Q_2 < 0$ are satisfied, $\langle ^j \phi \rangle$ increase with ζ or ξ increase because the second and third terms of the right side in (20) become positive. These conditions correspond to Reg_2. Therefore, the group state is organized through the division dynamics provided by the previous analysis.

IV. SIMULATION RESULTS AND DISCUSSION

A. Evaluation of adaptive division of labor

1) Simulation descriptions: This section demonstrates the effectiveness of the proposed method with simulation, which

TABLE II

SIMULATION CONDITION

n term	t	α	$({}^{j}x_i , {}^{j}x_1)$
1 term	$0 \leq t < 10$	0	not fixed
2 term	$10 \leq t < 20$	0.1	not fixed
3 term	$20 \leq t < 30$	0.1	not fixed
4 term	$30 \leq t < 40$	0	fixed to division
5 term	$40 \leq t < 50$	0	not fixed
6 term	$50 \leq t < 60$	0.1	fixed to not division
7 term	$60 \leq t < 70$	0.1	not fixed
8 term	$70 \leq t < 80$	0.1	not fixed
9 term	$80 \leq t < 90$	0.1	fixed to $(0, 1)$ at $t = 80$
10 term	$90 \leq t \leq 100$	0.1	fixed to $(1, 0)$ at $t = 90$

is the problem described in section 2. The variable values are calculated using the Euler method with $\Delta t = 0.001$ iteration. Firstly, we confirm how the evaluated value, strategy frequency ${}^{j}x_i$, and group mean fitness $\langle {}^{j}\phi \rangle$ change when the division is organized or disorganized. According to this simulation, the optimality and homeostasis of the homogeneous state or heterogeneous state is evaluated. To compare the $\langle {}^{j}\phi \rangle$ in homogeneous and heterogeneous state, α is fixed to 0 or 0.1 by reference to pre-simulation results. In addition, to evaluate the adaptive division of labor, that means $\langle {}^{j}\phi \rangle$ increasing, each ${}^{j}x_i$ are fixed under the table II. In the 1-3, 5, and 7, 8 terms, the normal simulation is carried out, and at $\alpha = 0$ or 0.1, the homogeneous or heterogeneous state is selected, respectively. In the 4 term, each ${}^{j}x_i$ are fixed to the values of the end of 3 term with only setting $\alpha = 0$. In this case, the strategy frequency is forcibly specialized to the heterogeneous state, although without interactions. In the 6 term, on the contrary, each ${}^{j}x_i$ is fixed to the values of the end of 5 term, setting only $\alpha = 0.1$. In this case, the group state is forcibly a homogeneous state, even though robots can interact. With results of the 4 term and 6 term, the adaptive division of labor is evaluated by increase and decrease of the group mean fitness. For the 9 term, the strategy frequency is fixed instantaneously to $({}^{j}x_1, {}^{j}x_2) = (0, 1)$ at the beginning of term $t = 80$. In the 10 term, in contrast, the strategy frequency is fixed instantaneously to $({}^{j}x_1, {}^{j}x_2) = (1, 0)$ at the beginning of the term $t = 90$. In ordinary circumstances, the group state converges to heterogeneity because of $\alpha = 0.1$. Through these operations, regarded as environmental disturbances, the stability and adaptive properties of division rate are evaluated.

2) Results: Figures 4(a) and 4(b) respectively depict the time evolution of ${}^{j}x_1$ and ${}^{j}x_2$ simulated on the condition table II. Figure 4(c) shows the average of $\langle {}^{j}\phi \rangle$ during each term. Comparison of the 4 term and 5 term shows that group mean fitness $\langle {}^{j}\phi \rangle$ in the 4 term. The division is forcibly organized in this term; it persists at low levels against the evaluated value in the 5 term. This result reflects the effectiveness of roles of ${}^{j}h_i$. This compression indicates that robots can not improve group performance with division that is not organized by appropriate interactions. The performance decrease is also readily apparent from eq. (20). Without the forcible heterogeneous state, the homogeneous state is ordinarily selected. Then, the Jacobian of the second and third terms in the right side of the equation must be positive,
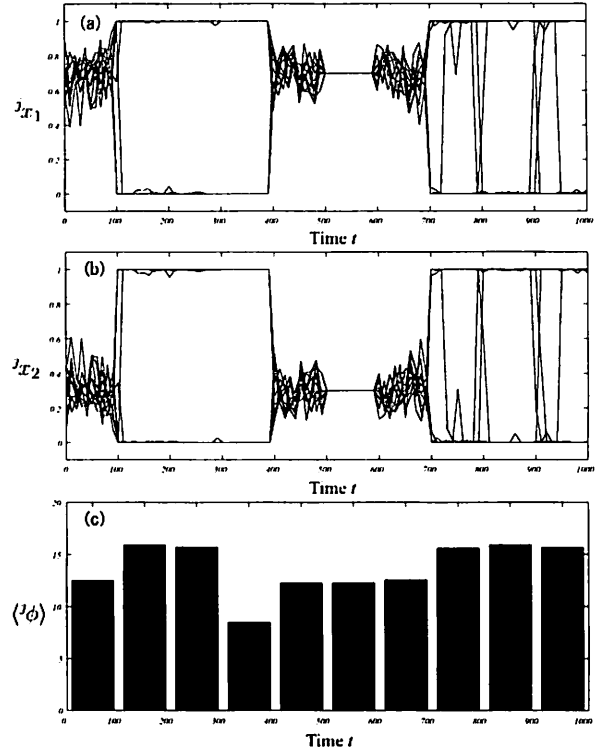

Fig. 4. Result of convergence property for environmental disturbances

the group mean fitness decreases because of $\zeta > 0$ and $\xi > 0$, which indicates a heterogeneous state. Comparison of the 3 term and 6 term shows that, for the evaluated value in the 6 term, the division is forcibly disorganized in this term; it persists at low levels against the evaluated value in the 3 term. This comparison indicates the improvement of performances with division. With explanation using eq. (20), without the forcible homogeneous state, a heterogeneous state is ordinarily selected. Then, the Jacobian of the second and third terms in the right side of the equation must be negative, the group mean fitness can not increase with $\zeta = 0$, $\xi = 0$, which means a homogeneous state. These two comparisons of results suggest that the proposed method has functions that are not only self-organized state but also reflective of an adaptive division of labor.

In addition, at $t = 80$ and $t = 90$, all of $({}^{j}x_1, {}^{j}x_2)$ are set to $(0, 1)$; and $(1, 0)$ is used as an assumption of some accidents in the 8 term and 9 term, such as the breakdown of a robot. Robots can recover and organize the ratio of division as it had been before. This result confirms the adequacy of the stability analysis; the proposed method has a homeostatic property. It is considered that the roles of ${}^{j}h_i$ are not only the exchange of food to be assimilated but also order formation of the heterogeneous state and regulation of the division rate.

B. Scalability against changes of a number of robots

1) Simulation descriptions: This subsection demonstrates the scalability of the number of robots with $m = 2$. All other conditions are identical to those of the simulation in
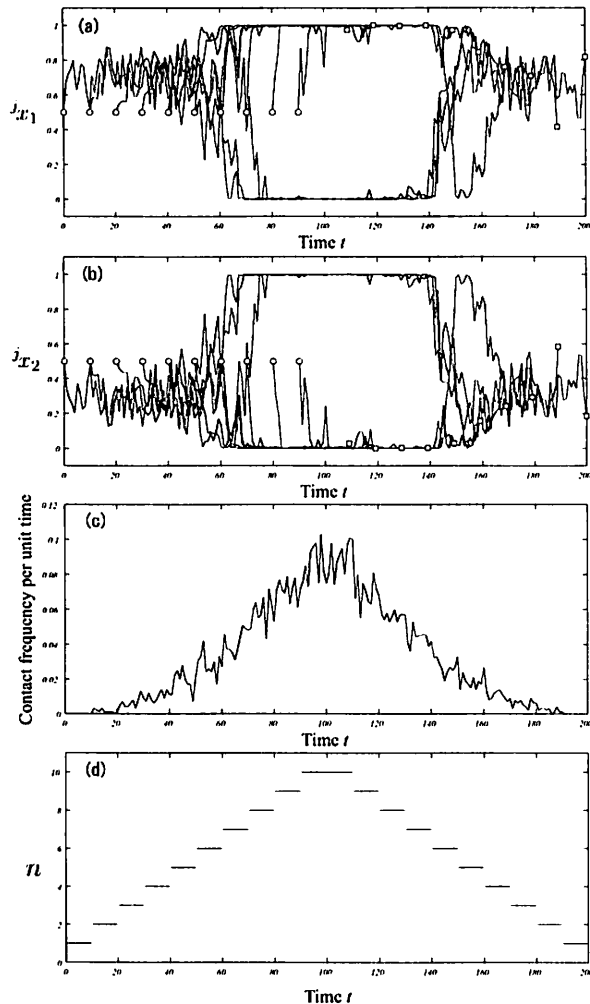
Fig. 5. Result of convergence property for variance of the number of robots

operated with only contact frequency. This is one reason that the adaptive division of labor is achieved by decentralized control. The group state is selected autonomously and driven to a homogeneous or heterogeneous state when α is determined.

C. Discussion

The theoretical result presented by eq. (20) gives useful information to understand the adaptively and optimality of division of labor. In proposed method, the division of labor is controlled through the phase transition, although transition does not generally guarantee the group performance improvement. The interactions are simple, but the adaptive division of labor is realized. The determination of whether a state of stabilization or destabilization pertains is described by the Jacobian, as shown in eq. (20). Its positive and negative values are directly linked to group fitness improvement. The controlled state, at least the problem of homogeneity or heterogeneity, already satisfies the condition of group performance improvement.

V. CONCLUSION AND FUTURE WORK

Herein, we proposed adaptive division of labor for robot group. Additionally, it was found through theoretical analyses that the group performance is certainly improved in the two-dimensional labor case, and that the dynamical behavior of both robots' and group states is observed through computer simulations. The future work includes the development of three or more dimensional division of labor method, and experiment by real robots.

VI. ACKNOWLEDGMENTS

previous subsection, except that n changes from 1 to 10. The j-th robot carries out the labor during $20(j - 1) \leq t \leq 20(11 - j)$ on the working space (cf. Fig. 5(d)). When the robot is put randomly into the working space, the strategy frequency is initialized as $(^j x_1, ^j x_2) = (0.5, 0.5)$. Results of this simulation underscore that the division of labor can be carried out with scalability of the number of robots, and depends on the contact frequency.

2) Results: Figures 5(a) and 5(b) respectively portray the time evolution of $^j x_1$ and $^j x_2$. The white circles and white squares indicate the values of $(^j x_1, ^j x_2)$ when the robot is put into and picked up from a working space. Figure 5(c) presents the time evolution of summation of contact frequency per time. The contact frequency increases (decreases) according to the increase (decrease) in the number of robots in the working space. When, $n > 5$, the heterogeneous state is organized because it is considered that the spatial density of robots increases, satisfying sufficient interaction for division. This result is straightforward. However, it indicates the asset effectiveness because the proposed method is mainly

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