Path and Viewpoint Planning of Mobile Robots with Multiple Observation Strategies

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***Abstract*— In this paper, we propose a new path and viewpoint planning method for a mobile robot with multiple observation strategies. When a mobile robot works in the constructed environments such as indoor, it is very effective and reasonable to attach landmarks on the environment for the vision-based navigation. In that case, it is important for the robot to decide its motion automatically. Therefore, we propose a motion planning method that optimizes the efficiency of the task, the danger of colliding with obstacles, and the accuracy and the ease of the observation according to the situation and the performance of the robots. We also introduce multiple landmark-observation strategies to optimize the motion depending on the number and the configuration of visible landmarks in each place.**

***Index Terms*— Path planning, Viewpoint planning, Mobile robot, Motion planning, Multiple observation strategies**

I. INTRODUCTION

In this paper, we propose a new path and viewpoint planning method for a mobile robot that has two active cameras according to the performance of the robot.

The robot navigation is a very important technology to execute various tasks. The navigation is usually executed while the robots move and estimate their positions and orientations by using the information from several sensors.

A dead-reckoning is a method that can estimate the robot position and orientation with internal sensor. However, the direction in which the robot travels, as well as its velocity depends on a lot of parameters such as floor structure, previous wheel positions and so on. Therefore, the error of estimated position and orientation is accumulated in proportion to the traveling distance and the recovery of the error is impossible only with internal sensors.

Therefore, external sensors are always utilized for the robot navigation. An image-based navigation by using cameras is one of the most popular methods. The robot can observe characteristic points in the environment and measure the relationship between these points and the robot itself in the image-based navigation. The characteristic point is always called "landmark". When the robots observe landmarks, the problems are how to measure the accurate position and orientation of landmarks, and where and which landmarks the robots should observe while there are multiple landmarks.

As to the former problem, there are a lot of studies that improve the accuracy of 3-D measurement, *i.e.*, [1]-[4]. However, there is a limit in accuracy when the robot always observes the same landmark regardless of the distance between the robot and the landmark.

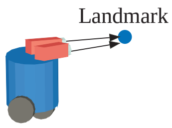
Therefore, the latter problem is very important for the robot navigation. This means that the robot must decide the path and choose the observing landmarks according to its position while it moves[5]-[8]. Komoriya *et al.* proposed the planning method of observing landmarks when there are multiple landmarks in the robot's field of view[5]. Nagatani and Yuta proposed the path and sensing point planning method. In this method, the cost function for the mobile robot's path that minimizes the risk of collision was settled, and the algorithm to find the optimal path and sensing points was constructed[6]. Navigation planning methods under the uncertainty are also proposed[9],[10].

On the other hand, the design of the optimal arrangement of artificial landmark is very important in the indoor environment[11]. Takeuchi *et al.* proposed a method to dispose artificial landmarks in the environment, and to navigate a mobile robot there[12]. The landmarks are disposed so that the robot can observe at least one landmark at any positions, and the robot plans the observation so that they can observe landmarks.

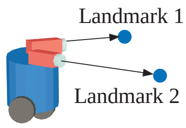
II. PURPOSE

Previous works consider the observing landmarks or viewpoints while there are several landmarks in the robot's field of view. However, these methods do not consider multiple observation strategies (Fig. 1). If the number of the camera is more than two and the number of visible landmarks is also more than two, the robot can observe landmarks in the several ways. For example, when two landmarks (landmark 1 and 2) are in the robot's field of view, the robot can observe one selected landmarks (landmark 1 or 2) with two cameras by using the stereo observation method. Besides the above-mentioned stereo observation, the robot can observe landmark 1 with the left camera and landmark 2 with the right camera.

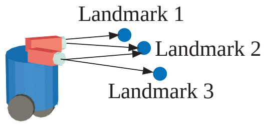
We have already proposed landmark observation method with multiple strategies[13]. In this method, the robot uses the information from two active cameras that can change their directions independently. They can observe landmarks with several types of the observation strategies to change



(a) Stereo observation.



(b) Two-landmark observation.



(c) Three-landmark observation.

Fig. 1. Three observation strategies.

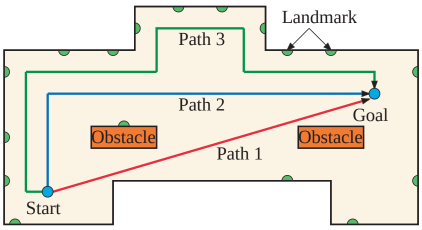


Fig. 2. Comparison of robot path.

the cameras directions. However, paths and viewpoints are given in [13].

Optimal path and viewpoint planning is necessary for the robot to execute works efficiently. At the same time, "optimal" path and viewpoints change according to the performance of the robot. Some previous works can plan the optimal paths and viewpoints of the robot [14]-[16]. However, they don't consider the relationship between optimal planning results and the performance of the robot explicitly. For example, Path 1 is the shortest path in Fig. 2, although the number of visible landmarks is few and the path is very close to obstacles. The length of Path 3 is very long, however, there are not a few visible landmarks along the path. Path 2 has intermediate profile of Path 1 and Path 3. The evaluations of these paths are shown in Table I. However, this is a qualitative evaluation. If the performance of the robot is very good and the dead-reckoning error is equal to zero, Path 1 can be the shortest and safe path. When the robot has good cameras and the dead-reckoning error is large, Path 2 or Path 3 may be good path. Therefore, "optimal" path and viewpoints depend on the performance of the robot. There are multiple evaluation methods such as high accuracy, path length, and safety. Evaluation methods also change when the performance changes.

Therefore, we propose a new path and viewpoint planning method for a mobile robot with multiple observation strategies according to the performance of the robot.

TABLE I
EVALUATION OF ROBOT PATH.

	Efficiency	Accuracy	Safety
Path 1	Very good	Bad	Bad
Path 2	Fair	Good	Good
Path 3	Bad	Very good	Fair

The accuracy of multiple observation strategies depends on the positions of landmarks that can be observed. Therefore, the robot must choose the optimal landmark-observation strategy to consider the number and the configuration of visible landmarks in each place. We analyze the errors of the landmark shape and the position in images. The evaluation of each observation strategy is executed to consider the accuracy of the observation when the image errors occur. The optimal observation strategy is defined as the one that contains the minimum error when there is the image error. The directions of cameras and the landmarks that are observed are decided to plan the optimal observation strategy by using the environment map that indicates the positions of the landmarks. The path and viewpoints are also planned by considering not only the accuracy of the optimal observation strategy but also the dead-reckoning error of the robot.

Our motion planning method is designed to guarantee that the robot never collides with obstacle. In other word, the path gives the highest priority to satisfy the safety. The robot selects the shortest path from the safe paths. It means that the efficiency of the task is optimized under the condition that the robot never collides with obstacle. After deciding the shortest path, the number of viewpoints is minimized. This is because it is desirable that the cameras equipped with the robot are utilized for the other uses of the landmark-observation, *e.g.*, detecting unknown objects, tracking moving objects, and so on.

III. Three Observation Strategies

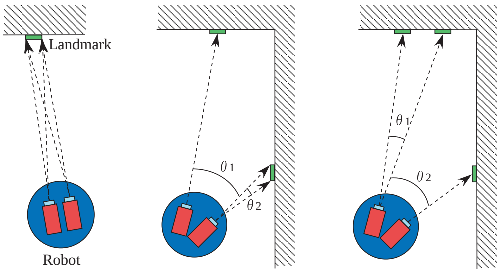
A. Problem Settlement

We make the assumptions for the planning of the robot navigation. The robot can move in all direction at any time and uses the information from two active cameras that can change their directions independently. The shape of the robot is expressed as the circle whose radius is R. The environment map that indicates the positions of walls, obstacles, and landmarks is also previously provided to the robot. All landmarks whose heights are same with those of robot's cameras are attached to the environment. The shape of the landmark is a circle and the radius of each landmark is constant. Each landmark can be distinguished with each other. All landmarks cannot be seen from the back side because they are attached on walls.

We develop three observation strategies: (a) stereo observation, (b) two-landmark observation, and (c) three-landmark observation.

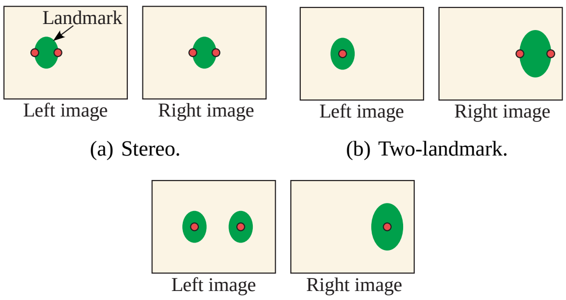
B. Stereo Observation

Te stereo observation can be executed when more than one landmark is inside the robot's field of view. The robot estimates its position and orientation with the triangulation (Fig. 3(a)). In this strategy, the 3-D positions of left and right ends of the observed landmark are measured with the information of disparities (Fig. 4(a)). The position and orientation of the robot in the world coordinate can be calculated from the coordinate value of two points.



(a) Stereo. (b) Two-landmark. (c) Three-landmark.

Fig. 3. Three observation strategies and landmarks.



(a) Stereo. (b) Two-landmark.

(c) Three-landmark.

Fig. 4. Acquired images in three observation strategies.

C. Two-Landmark Observation

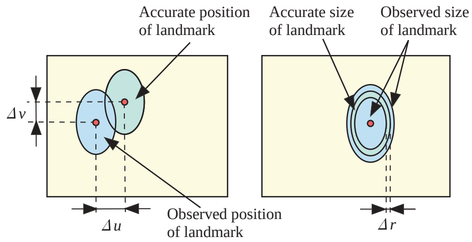
The two-landmark observation can be executed when more than two landmarks are inside the robot's field of view (Fig. 3(b)). Left and right ends of the nearer landmark and a center point of the distant landmark are extracted from two acquired images (Fig. 4(b)). The angles between extracted points (θ_1 and θ_2) can be calculated from the coordinate values in images. The position and the orientation of the robot in the world coordinate can be decided from these information, because there are three known points in the world coordinate and the relationships between these points and the robot is known.

D. Three-Landmark Observation

The three-landmark observation can be executed when more than three landmarks are inside the robot's field of view (Fig. 3(c)). The relationship between three landmarks and the robot is estimated from the coordinate value of the center of three landmarks in images (Fig. 4(c)). The position and the orientation of the robot can be decided as well as two-landmark observation method.

The accuracy of the estimated position and orientation become higher as compared with two-landmark observation method. This is because the distance between the extracted points in images is larger in three-landmark observation than in two-landmark observation.

In addition, the image noise at the edge of the landmark is generally larger than that at the center, because the coordinate value of the center position in the image coordinate can be obtained accurately by calculating the center of gravity of the pixels that belong to the landmark.



(a) Position error. (b) Size error.

Fig. 5. Image error.

E. Optimal Observation Strategy

The robot chooses the optimal landmark-observation strategy that can estimate its position and orientation precisely. The optimal strategy can be decided when the path and the position of the viewpoint is planned.

At first, the visible landmarks in each place are selected to consider the robot's field of view. The robot cannot observe landmarks when obstacles are between the robot and landmarks and cannot observe them from the back side.

Incidentally, the error of image such as quantization error always occurs. Then, the theoretical estimation errors of robot's position and orientation are calculated by considering the situation that the errors of landmark's position and size (shape) in images occur (Fig.5).

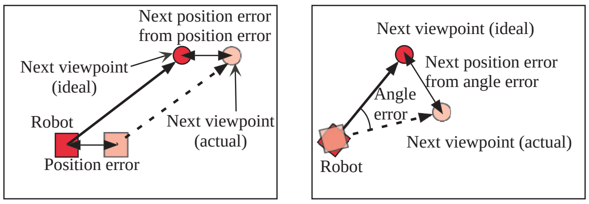
We assumed that the position error of the landmark's center point in the image is $(\Delta u, \Delta v)$. The size error of the landmark in the image Δr is also considered. It means that the observed landmark's position in the image may shift $(\pm\Delta u, \pm\Delta v)$ from the true position at the maximum. The observed radius may also shift $\pm\Delta r$ from the true size.

The robot estimates how the position and orientation errors occur in the world coordinate about all combination of visible landmarks when the errors occur in the images (image coordinates) of two cameras.

However, the position error and the orientation error are not compared directly because the dimensions of them are different from each other. The position error is expressed as the dimension of length, *i.e.*, [mm], and the orientation error is expressed as the dimension of angle, *i.e.*, [deg].

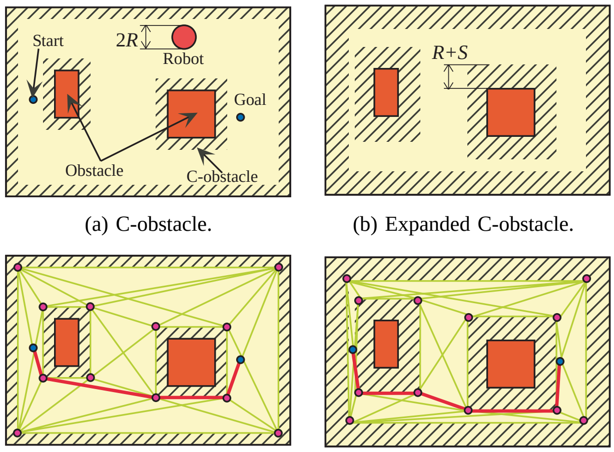
Therefore, we transform the orientation error (the dimension of angle) into the position error (the dimension of length). The total sum of the error when the robot moves at a certain distance while the position and orientation error occur is calculated (Fig.6). This means that the total error at the next time's position of the robot when it moves is the sum of the position error ($E_{pos,\max}$, Fig.6(a)) and the incorrect moving distance under the influence of the orientation error ($E_{ang,\max}$, Fig.6(b)). $E_{pos,\max}$ is the distance between the ideal robot position without error and the real position with error derived from position error. $E_{ang,\max}$ is the distance between the ideal robot position without error and the real position with error derived from orientation error. Therefore, $E_{pos,\max}$ and $E_{ang,\max}$ have the same dimension of length.

In this way, the estimated error of the robot position in



(a) Position error. (b) Orientation error.

Fig. 6. Position and orientation error of robot.



(a) C-obstacle. (b) Expanded C-obstacle.

(c) Visibility graph (S: small). (d) Visibility graph (S: large).

Fig. 7. Expanded C-obstacle and visibility graph.

the world coordinate that is caused by the image error is calculated. The optimal observation strategy is equal to the observation method that has minimum position estimation error.

The optimal observation strategy is decided as follows:

$$E_{\max}(p,m) = E_{pos,\max}(p,m) + E_{ang,\max}(p,m) \rightarrow \min, \quad (1)$$

where p is the present position, m is the moving distance, and $E_{\max}(p,m)$ is the total error when the robot move distance m from p.

The direction of two cameras is decided by selecting the optimal observation strategy in each place. Therefore, the navigation planning of the mobile robot can be executed.

IV. Path and Viewpoint Planning

A. Path Planning

The path of the robot is planned based on the visibility graph[17]. The C-obstacle (configuration obstacle) is usually generated by expanding the original obstacle at the width R that means a robot radius (Fig. 7(a)). In our motion planning method, original obstacles are expanded $R+S$ where S is the margin for safety (Fig. 7(b)). In this paper, we call $R+S(=D_s)$ the safety distance. When generating C-obstacles, shapes of their vertices are approximated with the polygons because of simplicity of computations.

The vertices of C-obstacles are connected with each other and a visibility graph is constructed. In this step, multiple visibility graphs are generated by changing S for optimal planning. In each visibility graph, the shortest path from a start position to a goal one is searched by Dijkstra algorithm. The second shortest, ..., the n-th shortest paths are searched in each visibility graph by a simple brute force method, too. This is because the robot cannot necessarily move along the shortest path due to its performance and the configuration of obstacles and landmarks, and several candidates of path must be prepared.

B. Viewpoint Planning

Viewpoint is planned about each path that is planned in the previous step. Viewpoint planning for arriving at the goal position safely is executed by considering the observation error of landmarks and dead-reckoning error.

The robot estimates its position and orientation by dead-reckoning while moving after observing landmark(s). If it moves long distance after estimating its position with landmark(s), the error of estimated position and orientation is accumulated in proportion to the traveling distance and there is a danger of colliding with obstacles. Therefore, the robot must estimate its position with landmark(s) frequently before position error is accumulated.

On the other hand, the direction change of cameras and image processing are needed for landmark observation. Therefore, it is good when the number of observing landmark(s) is small.

In this paper, the technique that can estimate its position and reach the goal safety without colliding with obstacles by the small number of observation is proposed.

Here, it is assumed that the robot observes landmark(s) where the distance between the present position p_s and the start position is m_s along the planned path. Let $D_{\max}(p_s,m)$ be the estimated maximum dead-reckoning error when the robot move distance m from p_s, $E_{\max}(p_s,m)$ be the estimated maximum error from the observation (1), and D_s (safety distance) means the maximum error of the robot position for not colliding with obstacles. The maximum distance m_{max} that the robot can move without observing landmarks is expressed as follows:

$$D_{\max}(p_s,m_{max}) + E_{\max}(p_s,m_{max}) \leq D_s. \quad (2)$$

Here, let p_g be the position whose distance from the start is $m_s + m_{p_s,\max}$. The path from p_s to p_g is divided into $n+1$ position, and we define m_i as the distance between the each divided position p_i and the start position of the path ($p_0 = p_s$, $p_n = p_g$).

When the next viewpoint from p_s is p_i, the next viewpoint from p_i must satisfy the following equation when the total number of observation becomes small (Fig. 8).

$$m_i + m_{p_i,\max} \rightarrow \max. \quad (3)$$

Therefore, the viewpoints can be decided one by one in the following way.

1) $m_{p_s,\max}$ and p_g that satisfy (2) are calculated when the robot observes landmarks at p_s.
2) The path from p_s to p_g is divided into $n+1$ position p_i.

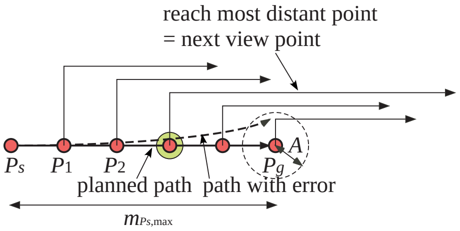


Fig. 8. Selection of view points.

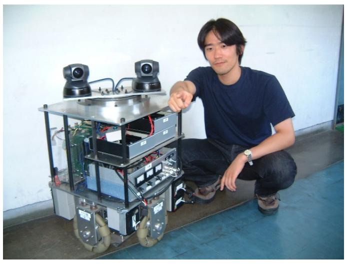


Fig. 9. Omni-directional mobile robot ZEN[18]

3) p_i that satisfies (3) is calculated, and p_i is regarded as the next viewpoint.
4) If $m_i + m_{p_i,\max}$ is smaller than the distance between the start position and the goal position of planned path, p_s is replaced with p_i and go to step 1). If it is large, the viewpoint planning finishes.

The optimal viewpoints, the optimal observation strategies in each viewpoint, the optimal observed landmark(s) and the direction of the cameras in each viewpoints can be planned in the above procedure.

V. Results of Motion Planning

At first, simulation for the real robot system is executed. All parameters are based on our constructed autonomous mobile robot system (Fig. 9). The focal length is 3000pixel, and the resolution of images is 640×480. The image error is set $\Delta u = \Delta v = \Delta r =$ 3pixel, and the dead-reckoning error is 15% of the moving distance.

Figure 10 shows the result of path and viewpoint planning. In this figure, circles mean viewpoints. The optimal observation methods are also decided with the motion planning. For example, two-landmark observation is selected when there are two visible landmarks around the goal position. Of course, when three landmarks can be observed, three-landmark observation is executed. From this result, the total path length is 3045mm, and the number of viewpoints is 15.

Next, we compare Robot R_1 whose dead-reckoning error and observation error are small and Robot R_2 whose errors are large, for evaluating the relationship between the planning result of the path and the viewpoints and the performance of the robots. As to R_1, we set $\Delta u = \Delta v = \Delta s =$ 1pixel, and the dead-reckoning error is 10% of the moving distance. As to R_2, we set $\Delta u = \Delta v = \Delta s =$ 3pixel, and the dead-reckoning error is 15%.

The distance between the start and the goal is short but there are few landmarks that can observed along the lower

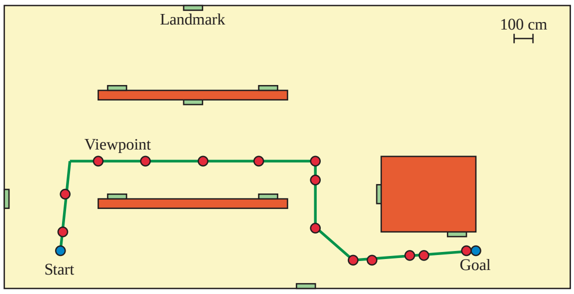


Fig. 10. Results of motion planning for real robot system.

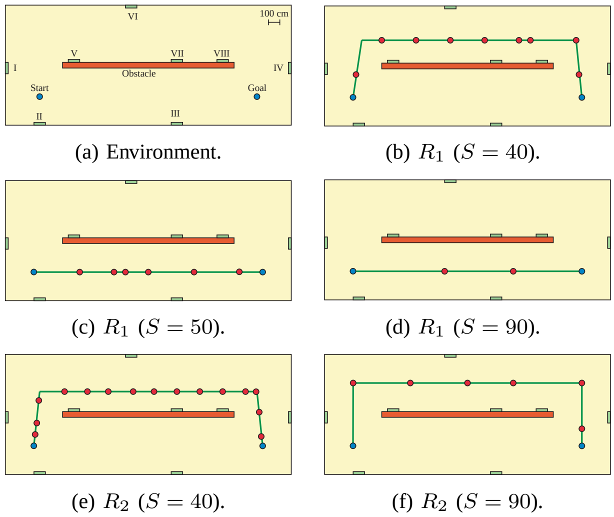


Fig. 11. Results of motion planning for comparison of robot performance.

path in Fig. 11(a). Therefore, the robot can reach the goal in the short time. On the other hand, the distance is long but there are a lot of landmarks along the upper path. Therefore, the robot can estimate its position with high accuracy along the upper path.

As the result of planning, R_1 can reach the goal along the lower path shown in Fig. 11(d), although R_2 cannot reach the goal along the upper path shown in Fig. 11(e). This is because there is possibility of colliding with obstacles along the lower path and it is planned that R_2 can reach the goal along the upper path. Of course, observation strategies can be selected for the numbers and configurations of the visible landmarks in all viewpoints.

From these results, it is shown that the three-landmark observation strategy is the best method in all situation if more than three landmarks can be seen. The accuracy of the two-landmark observation strategy depends on the angle between two visible landmarks. If the angle is large, the two-landmark observation strategy is adopted, and if it is small, the stereo observation strategy is selected as the optimal observation method.

To evaluate the results quantitatively, the comparisons of R_1 and R_2 paths are shown in the in Table II and III, respectively. About R_1, paths change drastically between $S = 40$ and $S = 50$. About R_2, paths change mildly because the robot does not have good performance, and cannot move along the lower path.

TABLE II

COMPARISON OF ROBOT R_1 PATH WHEN S CHANGES.

Safety distance	Path	Path length	Viewpoint
10	—	—	—
20	Upper	2793	27
30	Upper	2830	14
40 (Fig. 11(b))	Upper	2867	10
50 (Fig. 11(c))	Lower	2000	7
60	Lower	2000	5
70	Lower	2000	4
80	Lower	2000	4
90 (Optimal, Fig. 11(d))	Lower	2000	3

TABLE III

COMPARISON OF ROBOT R_2 PATH WHEN S CHANGES.

Safety distance	Path	Path length	Viewpoint
10	—	—	—
20	—	—	—
30	—	—	—
40 (Optimal, Fig. 11(e))	Upper	2867	16
50	Upper	2904	11
60	Upper	2943	11
70	Upper	2981	10
80	Upper	3020	8
90 (Fig. 11(f))	Upper	3060	7

From these results, it is shown that the optimal path, viewpoints, observation strategies can be planned according to the performance of the robot. In concrete terms, the robot with high performance (small dead-reckoning and observation error) can select the short path from the start to the goal, although there are few landmarks and this is a dangerous path. Contrarily, the robot with low performance selects the safe path in which a lot of landmarks can be observed and the accuracy of positioning is high, although the distance between the start and the goal is long.

VI. CONCLUSIONS

In this paper, we propose a new path and viewpoint planning method for autonomous mobile robots with multiple observation strategies. The robot estimates its position by observing landmarks attached on the environments with two active cameras. It chooses the optimal landmark-observation strategy depending on the number and the configuration of visible landmarks in each place. The optimal path, viewpoints, and observation strategies that minimize the estimation error of the robot position and the number of observation can be selected properly. The effectiveness of our method is shown through simulations.

As the future works, we have to decrease the computation time to find the k-shortest paths more efficiently[19]. It should be better that models of sensor and motion error are based on probabilistic ones, such as Kalman filters, particle filters, SLAM, and so on[20]-[22], when experiments with real robot system are performed.

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