# **Development of Pedestrian Behavior Model Taking Account of Intention**

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Abstract— In order for robots to safely move in human-robot coexisting environment, they must be able to predict their surrounding people's behavior. In this study, a pedestrian behavior model that produces humanlike behavior was developed. The model takes into account the pedestrian's intention. Based on the intention, the model pedestrian sets its subgoal and moves toward the subgoal according to virtual forces affected by other pedestrian and environment. The proposed model was verified through pedestrian observation experiments.

## I. INTRODUCTION

In human-robot coexisting environment, safety is one of the most important issues. In order for robots to safely coexist with humans, collision avoidance is of great importance. As an example from industry, pre-crash safety systems that can predict the collision by sensing the surrounding environment and prevent it before occurring are being introduced into many vehicles. In robotics studies, many researchers have been working on the collision avoidance [1], [2].

For intrinsic safety, of course, a function of emergency avoidance must be installed in robots. Not only that, obviating the risk of collision is very important for overall safety. In fact, when a person is in the environment including other people and human-controlled vehicles, its behavior must be affected by the existence of others and their behaviors. In order for robots to safely coexist in such an environment, therefore, they should not regard humans as just moving obstacles. The robots need to accurately predict surrounding human behaviors.

To predict human behaviors, a model that provides humanlike behaviors in accordance with surrounding environment is required. There have been several studies about pedestrian model since early times [3] and most of them have been done for crowd simulation of evacuation. Models that consider crowd as a fluid [3], [4] can handle pedestrian crowd, however they are not appropriate to predict individual pedestrian behavior. On the other hand, microscopic pedestrian models that handle individual pedestrian behavior have been proposed later. The microscopic models can be divided into two distinct groups. One is using a cellular automaton in a discrete space [5], [6], and another treats continuos motion in continuous space [7]–[10]. Models in the first group cannot produce humanlike behaviors because the simulated pedestrian can move only to the adjacent cells. In other words, although the models are microscopically described, the microscopic behaviors produced by the models are less humanlike. Models in the second group have capacity to produce humanlike behaviors because the motion of the simulated pedestrian is not unnaturally restricted. To predict pedestrian behaviors, therefore, the latter approach is preferable.

Social force model [7], [8] is one of the most popular and useful models. The model assumes that virtual forces act on a pedestrian from other pedestrians and environments and the pedestrian moves according to the resultant force of them. The social force model is one of the most popular models and it is to some extent useful for predicting pedestrian behaviors by a mobile robot [11]. Also there have been several improved versions of it. For example, Zanlungo *et al.* extended the model explicitly considering the prediction of collision [9]. Nevertheless, the existing models could not produce humanlike behaviors in some cases.

The reason seems that the intention of pedestrian was not considered in the models. In reality, pedestrians behave based not only on the information perceived from surrounding environment but also on their internal state (e.g. frustration) and intention. The objective of this study is to develop a pedestrian model that produces humanlike behaviors by considering the pedestrian's intention.

In section II, the proposed pedestrian model is described in detail. Section III presents experiments for identification of model parameters. In section IV, comparison between observed pedestrian behaviors and simulated behaviors produced by the proposed model is shown. Finally, we conclude this paper and refer the future plans in section V.

## II. MODELING OF PEDESTRIAN'S BEHAVIOR

# A. Basic concept of the model

Our pedestrian model is based on the social force model [7] for calculating low-level motion. The social force model considers four types of virtual forces acting on a pedestrian  $\alpha$  as follows (Fig. 1):

- Acceleration force towards a goal:  $f^0_{\alpha}$
- Repulsive force from other pedestrian  $\beta$ :  $f_{\alpha\beta}$
- Repulsive force from obstacle B:  $f_{\alpha B}$
- Attractive force from other pedestrian or place *i*:  $f_{\alpha i}$

In this study, the fourth one is omitted for simplicity and the pedestrian moves according to the resultant force of them.

$$\boldsymbol{f}_{\alpha} = \boldsymbol{f}_{\alpha}^{0} + \boldsymbol{f}_{\alpha\beta} + \boldsymbol{f}_{\alpha B} \tag{1}$$

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Fig. 1. Social force model



Fig. 2. Concept of intentions

As mentioned before, the social force model produces unnatural behaviors compared with real pedestrian. A possible cause of this is that the social force model does not change the pedestrian's goal according to the situation, whereas real pedestrians sometimes set a temporal goal as the situation demands.

In order to resolve this problem, in this study, we introduce a mechanism to create an appropriate subgoal according to the pedestrian's attempted behavior at the moment. Here, the term "behavior" means "avoiding a person coming towards me" or "following a person walking in front of me," etc. Pedestrian's behavior is not necessarily correspond to its intention. For example, though it wants to overtake a person in front of it while walking through a narrow aisle, it may think it cannot overtake him and decides to follow him for a while. While there are several behaviors/intentions pedestrians select, we deal with three types of typical and basic behaviors/intentions, such as *free walk, avoid*, and *follow* (Fig. 2).

When the pedestrian has a goal to reach, and on the way to its goal there is no other pedestrian and obstacle that may



Fig. 3. Overview of the proposed pedestrian model

affect its traveling, its intention is defined to be *free walk*. If there are some other people interfering with the pedestrian's way to goal, it will switch its intention to either *avoid* or *follow*.

In the proposed model, a pedestrian decides its intention based on its internal state and environmental situation. Based on the intention, it selects its behavior and sets a subgoal. After that, the model calculates the social forces acting on it according to the environmental situation and its subgoal. Finally, the model produces its motion and the motion affects its internal state. The overview of the proposed pedestrian model is shown in Fig. 3.

#### B. Intention transition and behavior selection

Since pedestrian's intention primarily controls its behavior, in the proposed model, there are limited variety of behavior available for each intention. If the pedestrian's intention is *free walk*, it only does *free walk*. Also, if its intention is *follow*, it only selects *follow* behavior. Our point here is if its intention is *avoid*, it has two behavior options, *avoid* and *follow* and selects appropriate one according to the prediction of the future situation. The relationship between intention transition and behavior selection is shown in Fig. 4.

1) Free walk: A concept of "warning area" is introduced here (Fig. 5). The concept comes from the idea that a pedestrian only concerns about other people who probably interfere with its traveling. The warning area of pedestrian  $\alpha$  is determined based on its desired direction  $e_{\alpha}$  and its field of view. Desired direction means the direction toward its goal and the warning area is provided ranges of d around the direction. Here, field of view is determined by the range R and view angle. For simplicity, the view angle is set to  $180^{\circ}$ . The warning area is defined as the intersection of field



Fig. 4. Intention transition and behavior selection



Fig. 5. Definition of warning area

of view and desired direction with ranges.

If there is no other pedestrian in the warning area of  $\alpha$ , its intention will be set to *free walk* and its motion will be calculated by Eq. (1).

If other pedestrian  $\beta$  penetrate the pedestrian  $\alpha$ 's warning area, intention of  $\alpha$  will switch to either *avoid* or *follow*.

• If the traveling directions of  $\alpha$  and  $\beta$  are roughly opposite (Eq. (2)),  $\alpha$ 's intention will switch to *avoid*.

$$\boldsymbol{v}_{\alpha} \cdot \boldsymbol{v}_{\beta} \le 0 \tag{2}$$

• If they move in basically the same direction (Eq. (3)),  $\alpha$ 's intention will be determined depending on the relative speed of  $\alpha$  and  $\beta$ .

$$\boldsymbol{v}_{\alpha} \cdot \boldsymbol{v}_{\beta} > 0 \tag{3}$$

- If their speed difference is relatively low or  $\alpha$  moves slower than  $\beta$  (Eq. (4)),  $\alpha$ 's intention will switch to *follow*.

$$\|\boldsymbol{v}_{\alpha}\| - \|\boldsymbol{v}_{\beta}\| < v_1 \tag{4}$$



Fig. 6. Creation of a subgoal for following behavior

On the other hand, if α moves much faster than β (Eq. (5)), α changes its intention to *avoid* for overtaking β.

$$\|\boldsymbol{v}_{\alpha}\| - \|\boldsymbol{v}_{\beta}\| \ge v_1 \tag{5}$$

Here,  $v_{\alpha}$  and  $v_{\beta}$  are the velocity of pedestrian  $\alpha$  and  $\beta$ , and  $v_1$  is the non-negative threshold value.

2) Follow: When pedestrian  $\alpha$ 's intention is follow, it tries to keep a suitable distance with its target pedestrian  $\beta$ . In the proposed model, we introduce a subgoal  $\gamma$ , which is set behind the pedestrian  $\beta$  (Fig. 6).  $r_{\gamma}$ , the position of  $\gamma$ , is calculated by the following equation:

$$\boldsymbol{r}_{\gamma} = \boldsymbol{r}_{\beta} - d_{\beta\gamma} \boldsymbol{e}_{\alpha}, \tag{6}$$

where  $r_{\beta}$  is the position of  $\beta$  and  $d_{\beta\gamma}$  is the suitable distance between  $\beta$  and  $\gamma$ .

In this case, the social force equation is modified. The acceleration force  $f_{\alpha}^{0}$  is replaced with  $f_{\alpha}^{\gamma}$ , the acceleration force towards the subgoal  $\gamma$ . Also the repulsive force from pedestrian  $\beta$  decreases from  $f_{\alpha\beta}$  to  $f_{\alpha\beta-}$ . As a result, the resultant force is described as follows:

$$\boldsymbol{f}_{\alpha}^{\text{follow}} = \boldsymbol{f}_{\alpha}^{\gamma} + \boldsymbol{f}_{\alpha\beta-} + \boldsymbol{f}_{\alpha B}$$
(7)

Following somebody intensifies the pedestrian's frustration which means the follower is getting frustrated of following and becomes eager to overtake the person it is following. Because of this reason, we introduce the frustration value into the pedestrian model. The difference between pedestrian  $\alpha$ 's desired speed and the speed of pedestrian  $\beta$  accumulates with time (Eq. (8)), and if the accumulated value exceeds  $\varepsilon_{\rm fr}$ , the threshold about frustration,  $\alpha$ 's intention will switch to *avoid*.

$$F_{\alpha\beta}(t) = \int_0^t |v_{\alpha}^{\text{desired}} - \|\boldsymbol{v}_{\beta}(t)\| |d\tau \qquad (8)$$

Here,  $F_{\alpha\beta}(t)$  is  $\alpha$ 's frustration value affected from  $\beta$  at t and  $v_{\alpha}^{\text{desired}}$  is the desired speed of  $\alpha$ .

If other pedestrians go out of the warning area of  $\alpha$ ,  $\alpha$ 's intention will switch to *free walk*.

3) Avoid: When pedestrian  $\alpha$ 's intention is avoid, two subgoal candidates are created at the both side of other pedestrian  $\beta$  (Fig. 7).

 $r_{\delta_L}$  and  $r_{\delta_R}$ , the positions of the subgoal candidates  $\delta_L$  and  $\delta_R$ , are calculated as follows:

$$\boldsymbol{r}_{\delta_L} = \boldsymbol{r}_{\beta} + d_{\delta 1} \boldsymbol{e}_{\alpha} + d_{\delta 2} \boldsymbol{e}_{\alpha}^{\perp},$$

$$\boldsymbol{r}_{\delta_R} = \boldsymbol{r}_{\beta} + d_{\delta 1} \boldsymbol{e}_{\alpha} - d_{\delta 2} \boldsymbol{e}_{\alpha}^{\perp},$$
(9)



Fig. 7. Creation of subgoal candidates for avoiding behavior

where  $d_{\delta 1}$  and  $d_{\delta 2}$  are longitudinal and lateral distance between pedestrian  $\beta$  and a candidate, respectively.  $e_{\alpha}^{\perp}$  is a unit vector normal to the desired direction  $e_{\alpha}$ .

After that, the proposed model evaluates the candidates based on prediction process. In the prediction process, the pedestrian model calculates the future situation until pedestrian  $\alpha$  reaches the subgoal candidates. At each time step, the model determines if other pedestrian penetrates  $\alpha$ 's warning area at the moment, and if it predicts that someone penetrates the area, it will judge the subgoal candidate destined for as bad. In this prediction process, the motion of pedestrian  $\alpha$ is based on the modified social force (Eq. (10)) and that of  $\beta$  is linear uniform motion.

If a subgoal candidate is not judged as bad, it will be regarded as a good subgoal candidate. Pedestrian  $\alpha$  decides the good candidate as the subgoal for avoiding behavior. If both candidates are good, pedestrian  $\alpha$  will choose nearer candidate from the current position as the subgoal. The social force is modified. In this case, the acceleration force is  $f_{\alpha}^{\delta}$ , which is from the selected subgoal  $\delta$ . The repulsive force from pedestrian  $\beta$  is  $f_{\alpha\beta-}$ , which is the same as that of *follow* behavior.

$$\boldsymbol{f}_{\alpha}^{\text{avoid}} = \boldsymbol{f}_{\alpha}^{\delta} + \boldsymbol{f}_{\alpha\beta-} + \boldsymbol{f}_{\alpha B}$$
(10)

On the other hand, if both candidates are bad,  $\alpha$  will be forced to select *follow* behavior. In this case, the social force is described as the following equation:

$$\boldsymbol{f}_{\alpha}^{\text{avoid-follow}} = \boldsymbol{f}_{\alpha}^{\theta} + \boldsymbol{f}_{\alpha\beta-} + \boldsymbol{f}_{\alpha B}, \qquad (11)$$

where  $f_{\alpha}^{\theta}$  is acceleration force from subgoal  $\theta$  for following  $\beta$ . This is similar to Eq. (7).  $r_{\theta}$ , the position of  $\theta$ , is calculated by the following equation:

$$\boldsymbol{r}_{\theta} = \boldsymbol{r}_{\beta} - d_{\beta\theta} \boldsymbol{e}_{\alpha}, \qquad (12)$$

where  $d_{\beta\theta}$  is the distance between pedestrian  $\beta$  and subgoal  $\theta$ .

As with *following* behavior, if other pedestrians go out of the warning area of  $\alpha$ ,  $\alpha$ 's intention will switch to *free walk*.

### **III. PARAMETER IDENTIFICATION**

As described in Section II, there are 9 parameters to be determined for the proposed model, such as d, R,  $v_1$ ,  $d_{\beta\gamma}$ ,  $v_{\alpha}^{\text{desired}}$ ,  $\varepsilon_{\text{fr}}$ ,  $d_{\delta 1}$ ,  $d_{\delta 2}$ , and  $d_{\beta\theta}$ . Moreover, we assumed that pedestrian's desired speed  $v_{\alpha}^{\text{desired}}$  is different in normal



Fig. 8. Experimental setting for parameter identification

and hurry condition. Therefore, we consider  $v_{\alpha}^{\rm normal}$  and  $v_{\alpha}^{\rm hurry}$  separately. As a consequence, 10 parameters have to be determined. In this study,  $v_1$  and  $\varepsilon_{\rm fr}$  were set to 0.5 m/s and 10 m, respectively, by try and error, because these values could not be estimated by observation. Besides these two parameters, 8 parameters were determined based on observation experiments.

#### A. Experimental setting and procedure

In the observation experiment, 6 healthy volunteers (aged 23–29 years; 5 men and 1 woman) were participated. The experiments were done in a square area measuring  $4 \times 13$  meters (Fig. 8). To measure participants' trajectories, we used two laser range finders (UTM-30LX, Hokuyo Automatic Co., Ltd.) and applied a simple tracking algorithm, which is adopted in [11]. The laser range finders were placed at the corners of the experimental area and the height of the sensors were 0.87 m. The measurement time interval was 100 ms.

Three experiments were carried out for parameter identification.

The first experiment was for estimating the desired speed  $v_{\alpha}^{\text{normal}}$  and  $v_{\alpha}^{\text{hurry}}$ . The participants walked straight from Start to Goal (Fig. 8-(a)) 6 times for each conditions.

The second experiment was to determine the parameters related to the warning area (R and d) and subgoal for avoidance ( $d_{\delta 1}$  and  $d_{\delta 2}$ ). In the experiment, the participants avoided an obstacle (stationary robot, which is 0.45 meters long, 0.45 meters wide, and 0.75 meters tall) while walking from Start to Goal (Fig. 8-(b)). In this case, the position of the obstacle were set at either 7.0 m, 8.0 m, or 9.0 m from the participants' start position. For each obstacle position, the participants walked 6 times.

TABLE I Identified parameters for each participant

Participant	A	В	С	D	Е	F
$v_{\alpha}^{\text{normal}}$ [m/s]	1.38	1.34	1.17	1.45	1.49	1.21
$v_{\alpha}^{\text{hurry}}$ [m/s]	2.24	1.87	1.98	2.01	2.29	1.74
R [m]	8.88	8.77	8.88	9.02	9.00	9.14
<i>d</i> [m]	1.04	1.26	0.73	0.86	0.58	0.84
$d_{\delta 1}$ [m]	0.26	-0.98	0.37	-0.08	1.60	-0.34
$d_{\delta 2}$ [m]	2.08	2.52	1.45	1.72	1.16	1.68
$d_{\beta\gamma}$ [m]	2.08	2.29	2.00	2.39	1.52	2.06
$d_{\beta\theta}$ [m]	1.04	1.15	1.00	1.19	0.76	1.03

The third experiment was to determine the parameters related to the subgoal for follow  $(d_{\beta\gamma} \text{ and } d_{\beta\theta})$ . In the experiment, the participants followed the robot (ZEN, Ritecs Inc.), which was also used as the obstacle in the second experiment, from Start to Goal (Fig. 8-(c)). The robot was initially placed either 3.0 m, 4.0 m, or 5.0 m from the participants' start position, and moved straight with speed of either 0.3 m/s, 0.5 m/s, or 0.8 m/s. The participants started following the robot at the moment the robot started to move. In the experiment, the participants walked 3 times for each combination of the robot's initial position and speed.

## B. Experimental results

In the first experiment, we eliminated the data of the first and last 2 seconds, in which the participants accelerated or decelerated their walking speed, and calculated the average speed to estimate the desired speed of each participant.

From the second experiment, the parameters R and d of the warning area is calculated by measuring the maximum longitudinal avoiding distance, which is estimated from the moment of changing direction, and the maximum lateral avoiding distance, respectively.  $d_{\delta 1}$  was calculated from the longitudinal distance when the lateral difference of the participant and the obstacle was the largest.  $d_{\delta 2}$  was estimated by doubling the parameter d.

In the third experiment, as with the case of the first experiment, the data of the first and last 2 seconds were eliminated and the distance between the robot and the participant was calculated to determine  $d_{\beta\gamma}$ .  $d_{\beta\theta}$  was assumed to be a half of the distance  $d_{\beta\gamma}$ .

Based on the three experiments, we obtained the eight parameters for each participant as shown in Table I.

# IV. COMPARISON BETWEEN SIMULATED AND OBSERVED PEDESTRIAN BEHAVIORS

## A. Pedestrian observation experiment

To evaluate the proposed model, the behaviors produced by the model was compared with the observed pedestrian behaviors. In the observation experiments, the participants and the measurement system were the same as Section III. The experiments were done in three situations.

Common in the three experimental scenarios, a participant (denoted  $\alpha$  in Fig. 9) were told to start walking from Start to Goal (denoted G in Fig. 9). We have another pedestrian in these experiments who is a cooperator (denoted  $\beta$  in Fig. 9). The cooperator was required to walk to either goal L,



Fig. 9. Experimental setting for evaluation

*C*, or *R*. In each trial, the goal was instructed to  $\beta$  by the experimenter, and the order of the goals were quasirandomized. Here, the participant  $\alpha$  could not know the goal of  $\beta$  beforehand. The cooperator  $\beta$  was asked to walk to each goal three times, which means there are 9 trials performed for each participant  $\alpha$  in each situation.

The first scenario was that participant  $\alpha$  walked to the goal while passing cooperator  $\beta$  who walked straight in opposite direction (Fig. 9-(a)).

The second scenario was that participant  $\alpha$  walked in a hurried pace to the goal, and cooperator  $\beta$  walked with normal speed to the instructed goal (Fig. 9-(b)). Different from the first scenario, the walking directions of  $\alpha$  and  $\beta$ were the roughly same.

The instruction to participant  $\alpha$  and cooperator  $\beta$  in the third scenario was the same as that in the second scenario. The third scenario was different from the second one in its environmental setting. There were obstacles as shown in Fig. 9-(c) and, because of this, there was a bottleneck in the experimental environment. Therefore, participant  $\alpha$  could not overtake  $\beta$  in the bottleneck area.

# B. Pedestrian behavior simulation

The proposed pedestrian model was implemented and simulated with C++ and OpenGL on Mac OS 10.6.8. In the simulation, the environmental information and observed cooperator  $\beta$ 's trajectory were input, and a simulated agent moved according to the proposed model.

In order to evaluate the proposed pedestrian model, we not only calculated the difference between the observed participants' trajectories and the simulated agents' trajectories, but also compared with the social force model [7].



Fig. 10. An example of the results in the first scenario



Fig. 11. Comparison of the proposed model and social force model

# C. Experimental results

Figure 10 shows an example of the experimental results in the first scenario. The green dashed line is an observed trajectory of the cooperator, and the blue dashed line is an observed trajectory of the participant. In this case, the cooperator's goal was R (Fig. 9-(a)), and the participant swerved to the right to avoid the cooperator. Simulated trajectories of the proposed model and social force model are the red solid line and the yellow dotted line, respectively. As can be seen in the figure, the simulated agent based on the social force model abruptly and unnaturally changed its direction in the middle of its trajectory. On the other hand, the proposed model could calculate a natural and smooth trajectory similar to the observed participant's trajectory.

Figure 11 shows the average difference between the observed participants' trajectory and the trajectories of the pedestrian models to compare the proposed model with the social force model.

Applying the proposed pedestrian model, the mean trajectory errors of the scenario 1, 2, and 3 were 0.43 m, 0.58 m, and 0.53 m, respectively. On the other hand, the mean errors using the social force model were 1.10 m, 1.08 m,

and 0.67 m. The mean trajectory errors were tested using paired *t*-test. There were significant differences between the proposed model and the social force model for the three scenarios (p < 0.01). Based on the results, it can be said that the proposed pedestrian model can produce more humanlike behaviors than the social force model.

#### V. CONCLUSION

In this study, we have developed a pedestrian behavior model considering pedestrian's intention. The proposed model can successfully simulate pedestrians' behaviors with results more humanlike than the social force model.

For future works, the proposed pedestrian behavior model will be applied to a large number of pedestrians. In this study, we omitted the attractive force from other persons or places and limited the number of pedestrian's intentions to three for simplicity. In the real world, however, people are sometimes attracted by other people, objects, places, etc. and they have much more intentions. Therefore the attractive force and other intentions should be considered in the next step. Furthermore, we will apply our proposed pedestrian model to various vehicles, such as cars and robots for prediction of their surrounding pedestrians' behaviors.

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