Iterative Energy Shaping of a Ball–Dribbling Robot

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Abstract: This paper deals with the problem of controlling a ball–dribbling robotic system. In particular, a novel hybrid controller which combines elements of passivity and iterative learning control theories has been developed. Numerical simulations have been performed to prove the validity of the proposed solution. Results confirmed the effectiveness of the developed methodology.

Keywords: robot control, hybrid systems, port–Hamiltonian systems, passivity–based control, iterative learning control.

1. INTRODUCTION

Modeling, analysis and control of mechanical systems with impacts is an interesting and open problem which attracts the attention of a wide range of researchers, from physicists and mechanical engineers to specialists in control and automation [Brogliato 1999, Stronge 2018]. The interaction between continuous and discrete–time dynamics arises, for instance, while considering the behavior of a mechanical system in presence of impacts, as its dynamics cannot be represented only by means of differential equations. The theory of *hybrid dynamical systems* (HDS) is the formalism used to accurately describe this peculiar phenomena. Overviews of this framework are given by Van Der Schaft and Schumacher 2000, Haddad et al. 2006. In particular, the most general modeling approach is the one of *hybrid inclusions* developed in recent years [Goebel et al. 2009].

When dealing with modelling and control of physical systems, one of the most popular state–of–the–art approaches is the the *port-Hamiltonian* (PH) theory [Secchi et al. 2007, Van Der Schaft et al. 2014]. PH systems provide a framework which can be employed to model physical systems from an energetic point of view, explicitly capturing the phenomena of energy storage, energy dissipation and energy routing. In this perspective a controller can also be thought as a dynamical system interconnected with the plant and exchanging energy with it. This idea led to the definition of *energy shaping* [Ortega et al. 2001], which represent a fruitful application of passivity-based control (PBC).

In this paper we present a novel method for controlling a *ball–dribbling* robot. A representation of the system is given in Figure 1. This model falls in the framework of *impact mechanics*. Several works attempted to address the control problem for some prototype examples of this class of systems, e.g. Sanfelice et al. 2007, Tian et al. 2013, Müller et al. 2011. In particular, the ball–dribbling problem has been considered by Bätz et al. [2010]. In their work, authors assumed the absence of any viscous friction effect, the exact knowledge of the ball's mass and the estimation of the impacts' restitution coefficients. While compared to the previous work, our method relaxes all the assumption by considering viscous frictions and it does not require the knowledge of any parameter characterizing the ball's dynamics.

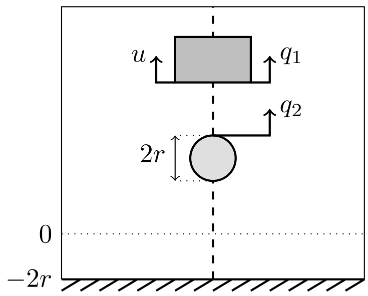

Fig. 1. One-dimensional ball–dribbling robotic system. The position of the robot (rectangle), is represented by the variable q_1 while u is an input force applied to it. Moreover, the position of the ball (circle), of radius r, is represented by q_2.

The novel control technique proposed in this paper attempts at casting energy shaping in a learning context. By "learning", here it is intended that the control law is adjusted on the basis of previous iterations. In chain, the concept of iterations, or trials, arises naturally in this context: within the class of systems that are considered, the discontinuous dynamics separating the flows can be used to implicitly recognize trials without introducing additional structure.

Numerical simulations, performed to steer the output of such a system along a periodic reference, successfully demonstrate the efficacy of the proposed approach. In particular, in the simulation experiments, the achieved control task is the periodic bouncing of the ball by means of impacts at a constant and prescribed height.

This paper is organized as follows: Section 2 introduces hybrid systems, port–Hamiltonian systems and the basic assumptions of this work. In Section 3 the model of the ball–dribbling robot is constructed. The proposed control paradigm, the *iterative energy shaping control*, is then employed to control the system in Section 4. Numerical simulations are provided to prove the effectiveness of the proposed control scheme. Finally, conclusion and future work are drawn in Section 6.

2. PRELIMINARIES

2.1 Notation

The set $\mathbb{R}$ is the the set of real numbers. The origin of $\mathbb{R}^n$ is $\mathbb{0}_n$. Let $\mathcal{H} : \mathbb{R}^n \to \mathbb{R}$ be a differentiable function and let $\partial\mathcal{H} \in \mathbb{R}^n$ be its transposed gradient, i.e. $\partial\mathcal{H} \triangleq (\nabla\mathcal{H})^\top \in \mathbb{R}^n$. Given two vectors $u,\ v \in \mathbb{R}^n$, let $(u, v) \triangleq [u^\top, v^\top]^\top$. Furthermore, $\text{diag}(v)$ denotes the diagonal matrix whose diagonal is v. The notation x^+ indicates the next value of the quantity x after a discrete–time event.

2.2 Background

Hybrid dynamical systems They represent a wide class of systems in which continuous time and discrete time dynamics interacts. This class of systems are often modeled through constrained *hybrid inclusions* [Goebel et al. 2009]. In this paper we will consider their single–flow specialization, in the form

$$\begin{cases} \dot{x} = f(x, u) & (x, u) \in \mathcal{C} \times \mathcal{U} \\ x^+ \in \mathcal{G}(x) & x \in \mathcal{D} \\ y = h(x) \end{cases} \tag{1}$$

with state $x \subseteq \mathbb{R}^n$, input $u \in \mathcal{U} \subseteq \mathbb{R}^m$ and output $y \in \mathcal{Y} \subseteq \mathbb{R}^m$. $f : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth vector field, $\mathcal{G} : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is a set-valued mapping and $\mathcal{C}, \mathcal{D}$ are subsets of $\mathbb{R}^n$ with. Let us call $\mathcal{C}$ the *flow set*, f the *flow map*, $\mathcal{D}$ the *jump set* and $\mathcal{G}$ the *jump map*.

Port-Hamiltonian systems The classical formulation of a finite-dimensional port-Hamiltonian system is:

$$\begin{cases} \dot{x} = [J(x) - R(x)]\,\partial\mathcal{H} + G(x)u \\ y = G^\top(x)\,\partial\mathcal{H} \end{cases} \tag{2}$$

where $J(x) = -J^\top(x)$ represents power preserving interconnections, $R(x) = R^\top(x) \geq 0$ represents dissipative effects and $G(x) \in \mathbb{R}^{m\times n}$ describes the way in which external power is distributed into the system. In general, $x \in \mathcal{X}$ being $\mathcal{X}$ an n dimensional manifold while $\mathcal{U}$ is a vector space. Since the output is a power variable dual to the input [Van Der Schaft et al. 2014], $\mathcal{Y} = \mathcal{U}^*$. The smooth map $\mathcal{H} : \mathcal{X} \to \mathbb{R}$ is denoted as *Hamiltonian function.* Note that PH systems are passive, i.e.

$$\dot{\mathcal{H}} \leq y^\top u$$

From now on let $F(x) \triangleq J(x) - R(x)$. Moreover, for compactness of notation, we will often omit the dependence on x of most of the aforementioned functions.

2.3 Basic assumptions for the ball–dribbling robot

Indeed, the ball–dribbling robotic system of Fig. 1 falls in the class of models of the form (1). In fact, the system presents a single flow, i.e. both the ball and the robot are "flying" and several jump, i.e. the *ball–floor* and *ball–robot* impacts. In order to develop an energy based control scheme for the ball–dribbling robot, the flow and output of the system will be described in port–Hamiltonian form, i.e.

$$\begin{cases} \dot{x} = F\,\partial\mathcal{H} + Gu & (x, u) \in \mathcal{C} \times \mathcal{U} \\ x^+ \in \mathcal{G}(x) & x \in \mathcal{D} \\ y = G^\top\,\partial\mathcal{H} \end{cases} \tag{3}$$

3. MODEL OF THE BALL–DRIBBLING ROBOT

3.1 Port-Hamiltonian Model of the Flows

Let q_1 be the position of the robot, q_2 the position of the ball and $p_1 \triangleq m_1\dot{q}_1$, $p_2 \triangleq m_2\dot{q}_2$ the momenta of the robot and the ball respectively. The state-space model of the system is:

$$\begin{cases} \dot{q}_1 = \dfrac{1}{m_1}p_1, & \dot{p}_1 = -m_1\gamma - \dfrac{\beta_1}{m_1}p_1 + u \\ \dot{q}_2 = \dfrac{1}{m_2}p_2, & \dot{p}_2 = -m_2\gamma - \dfrac{\beta_2}{m_2}p_2 \end{cases} \tag{4}$$

being γ the gravitational acceleration (i.e., $\gamma = 9.81m/s^2$), and $\beta_1,\ \beta_2 > 0$ the viscous friction coefficients. Let $q \triangleq [q_1, q_2]^\top$, $p \triangleq [p_1, p_2]^\top$ and $x \triangleq [q^\top, p^\top]^\top$. The system can be expressed in the canonical PH form (2) defining the Hamiltonian function as the total energy, i.e. $\mathcal{H}(q, p) \triangleq \frac{1}{2}p^\top M^{-1}p + V(q)$ where $M = \text{diag}([m_1, m_2])$ and $V(q) = \gamma[m_1, m_2]q$. Then, the PH dynamics are defined by $F = J - R$ being J a skew–Hamiltonian matrix and $R \triangleq \text{diag}[0, 0, \beta_1/m_1, \beta_2/m_2]$. Finally, the robot velocity is picked as output of the system, i.e.

$$G = [0\ 0\ 1\ 0]^\top \Rightarrow y = G^\top\,\partial\mathcal{H} = \frac{1}{m_1}p_1 = \dot{q}_1$$

guaranteeing passivity with the forcing term u as power conjugated input. Indeed the flow set $\mathcal{C}$ is

$$\begin{aligned} \mathcal{C} = \{x :\ q_1 \geq q_2 \geq 0\} \setminus [\{x :\ q_2 = 0 \wedge p_2 < 0\} \cup \\ \cup\{x :\ q_1 = q_2 \wedge (p_1p_2 < 0 \vee m_2p_1 < m_1p_2)\}] \end{aligned}$$

Remark 1. The behavior of the robot does not influence the flow of the ball, as can also be derived by physical considerations. However, the dynamics of the overall hybrid system will be coupled during jumps: the robot can influence the ball's motion (and vice versa) trough impacts.

3.2 Model of the Impacts

Considering the ball-dribbling robot, discontinuities of the system's state may happen in two situations: during the collision between ball and the floor or the one between the robot and the ball. Here, collisions are considered partially inelastic while both the robot and the ball are modeled as rigid bodies.

Ball-Floor Collisions
The collision of the ball with the ground will causes a sudden change of the ball's momentum:

$$p_2^+ = -c_g p_2$$

where $c_g \in (0,1)$ is the *ball-ground restitution coefficient.* Therefore, the resulting jump map is

$$x^+ = g_1(x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -c_g \end{bmatrix} x$$

The jump happens if $x \in \mathcal{D}_1$, where the jump set $\mathcal{D}_1$ is the following sub-manifold of the state space: $\mathcal{D}_1 = \{x : \; q_2 = 0 \wedge p_2 < 0\}$.

Robot-Ball Collisions
During the collisions between the robot and the ball, both the robot and ball momenta will change discontinuously as follows. The conservation law of the total momentum yields:

$$p_1^+ + p_2^+ = p_1 + p_2$$

Considering the partial inelasticity of the impacts:

$$\frac{p_1^+}{m_1} - \frac{p_2^+}{m_2} = -c_i \left(\frac{p_1}{m_1} - \frac{p_2}{m_2} \right)$$

where $c_i \in (0,1)$ is the *robot-ball restitution coefficient.* It follows that

$$\begin{bmatrix} p_1^+ \\ p_2^+ \end{bmatrix} = \begin{bmatrix} p_1 - \mu(m_2 p_1 - m_1 p_2) \\ p_2 + \mu(m_2 p_1 - m_1 p_2) \end{bmatrix}, \quad \mu = \frac{c_i + 1}{m_1 + m_2}$$

Thus, the corresponding jump map is

$$x^+ = g_2(x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 - \mu m_2 & \mu m_1 \\ 0 & 0 & \mu m_2 & 1 - \mu m_1 \end{bmatrix} x$$

The robot-ball collision happens if $x \in \mathcal{D}_2$, where the jump set $\mathcal{D}_2$ is: $\mathcal{D}_2 = \{x : \; q_1 = q_2 \wedge (p_1 p_2 < 0 \vee m_2 p_1 < m_1 p_2)\}$. The overall jump set $\mathcal{D}$ is defined as $\mathcal{D} = \mathcal{D}_1 \cup \mathcal{D}_2$ and the resultant set-valued mapping of the jumps is $\mathcal{G} \triangleq \{g_i : x \in \mathcal{D}_i \Rightarrow x^+ = g_i(x), \; i = 1, 2\}$. The final hybrid system can be then written in the form (3). It is easy to verify, that the Hamiltonian function decreases during jumps, i.e. $\mathcal{H}(g(x)) \leq \mathcal{H}(x) \quad \forall g \in \mathcal{G}, \; x \in \mathcal{D}$. Note that the state-space manifold $\mathcal{X}$ is: $\mathcal{X} \triangleq \mathcal{C} \cup \mathcal{D} = \{x : \; q_1 \geq q_2 \geq 0\}$.

4. DRIBBLING CONTROL

Let us consider the control task of continuously hitting the ball such that it reaches, at every cycle, the same maximum height $q_{2,max}^*$. In order to address this control problem, it is possible to design an hybrid controller with two modes, i.e., a *wait mode* (S_w) and a *hit mode* (S_h). In the wait state, the robot must stay at a constant height above the ball, to overcome any interference between its motion and the one of the ball and, at the same time, stay close enough to the ball to hit it quickly at the right time. Then, in the hit state, the controller must move the robot toward the ball so that the exchanged impulse during the impact would lead the ball to come back to the desired peak $q_{2,max}^*$. In particular, the system would enter in the hit state whenever the ball reaches the peak of its bounce and switch back to the wait mode immediately after the impact between the two bodies. In both modes, we would like to exploit the passivity–based control theory. Besides, the system is under–actuated and the flows are decoupled. Therefore, it is impossible to shape the total energy of system setting its minimum in a desired configuration different from the origin. Although it is not possible to modify the energy of the ball, it is however possible to partially shape the Hamiltonian, i.e. the part relative to the robot. If $\mathcal{H}(x) = \mathcal{H}_1(q_1, p_1) + \mathcal{H}_2(q_2, p_2)$, it is possible to obtained a desired-shape Hamiltonian $\mathcal{H}^*(x) = \mathcal{H}_1^*(q_1, p_1) + \mathcal{H}_2(q_2, p_2)$ allowing to bring the robot in a desired configuration q_1^*. This might be achieved through an *energy–balancing passivity–based controller* [1] [Ortega et al. 2001, 2008, Secchi et al. 2007]:

$$\begin{aligned} u &= \beta(x) + v = \beta(x) - k_d y \\ &= \frac{\partial V_1(q_1)}{\partial q_1} - k_p(q_1 - q_1^*) - k_d \dot{q}_1 \\ &= \gamma m_1 q_1 - k_p(q_1 - q_1^*) - k_d \dot{q}_1 \end{aligned}$$

As pointed out before, the controller has two separate modes and the control parameters k_p, k_d, q_1^* should be changed during the state transitions. For this reason, let us collect the control parameters in a vector $\omega = [k_p, k_d, q_1^*]^\top$ and consider it as part of the state vector. The augmented model of the controlled system can be then rewritten as follows:

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} F\partial\mathcal{H}^* + Gv(x,\omega) \\ 0 \end{bmatrix} & (x, \omega, v) \in \mathcal{C} \times \Omega \times \mathcal{U} \\ \begin{bmatrix} x^+ \\ \omega^+ \end{bmatrix} \in \mathcal{G} \times \Lambda & (x, \omega) \in \mathcal{D} \times \Omega \\ y = G^\top \partial\mathcal{H}^* \end{cases} \tag{5}$$

where Ω is the space of admissible parameters values and Λ is the jump set–valued mapping of the control parameters. The shaped Hamiltonian $\mathcal{H}^*(x, \omega)$, results to be

$$\mathcal{H}^*(x, \omega) = \frac{1}{2} p^\top M^{-1} p + \frac{1}{2} k_p (q_1 - q_1^*)^2 + \gamma m_2 q_2$$

Notice that the choice of the control parameters influences both the closed-loop energy $\mathcal{H}^*$ (with k_p and q_1^*) and the output feedback v (with k_d). Let us define a jump set $\mathcal{D}_3$ corresponding to the sub–manifold of the state–space where the ball reaches the peak $q_{2,max}$ of its bounce: $\mathcal{D}_3 = \{x : \; p_2 = 0\}$. It is clear that during the time evolution of the system, the state will cyclically enter in $\mathcal{D}_3$ and therefore the controller will periodically switch to $\mathcal{S}_h$ where the robot moves toward the ball until they collide, i.e. $x \in \mathcal{D}_2$, when the controller switches back to $\mathcal{S}_w$ where the robot waits above the ball at a distance δ. The jumps maps resetting the control parameters are the following:

$$\begin{aligned} \omega^+ &= \nu_3(x, \omega) = [k_{p,h} \;\; k_{d,h} \;\; q_2]^\top & x &\in \mathcal{D}_3 \\ \omega^+ &= \nu_2(x, \omega) = [k_{p,w} \;\; k_{d,w} \;\; q_2 + \delta]^\top & x &\in \mathcal{D}_2 \\ \omega^+ &= \nu_1(x, \omega) = \omega & x &\in \mathcal{D}_1 \end{aligned}$$

Therefore, $\Lambda \triangleq \{\nu_i : x \in \mathcal{D}_i \Rightarrow \omega^+ = \nu_i(\omega, x), \; i = 1, 2\}$. Fig. 2 shows the finite–state machine representing the controller. When $x \in \mathcal{D}_3$, the state of the system does not jump, i.e. $x^+ = g_3(x) = x$ if $x \in \mathcal{D}_3$. The set-valued mapping $\mathcal{G}$ is redefined considering g_3.

Remark 2. The behavior of the system strongly depends on the choice of the control parameters. Furthermore,

[1] In this case q_1^* becomes a strict minimum of $\mathcal{H}_1^*$ and asymptotic stabilization must be achieved with $p_1 = 0$ at steady–state

unless a solution of (5) is derived, it is not possible to find analytically two sets of control parameters (one for S_w and one for S_h) which solve the control problem, i.e. ensuring that the ball bounces continuously reaching each time the desired peak $q^*_{2,max}$.

For the reasons stated in previous remark, a new paradigm of energy based control, which combines the traditional energy shaping approach with a basic form of iterative learning control: the *iterative energy shaping*, has been introduced.

Definition 3. (Iterative Energy Shaping Control).
First, let us define some further control parameters which are needed for the design. Let ξ be a counter of the number of cycles of the system, i.e., the number of complete bounces of the ball. Let initialize ξ to one. Let $e(\xi) = q^*_{2,max} - q_{2,max}$ computed when $x \in \mathcal{D}_3$ be the tracking error of the iterative energy shaping control loop. Thus, the control parameters vector can be redefined as $\omega \triangleq [k_p, k_d, q^*_1, \xi]^\top$. The iterative energy shaping control law is defined by means of the following control parameters jump maps:

$$\omega^+ \triangleq \nu_3(x,\omega) = [\sigma\varphi_\xi(e) \;\; k_{d,h} \;\; q_2 \;\; \xi+1]^\top \qquad x \in \mathcal{D}_3$$
$$\omega^+ \triangleq \nu_2(x,\omega) = [k_{p,w} \;\; k_{d,w} \;\; q_2+\delta \;\; \xi]^\top \qquad x \in \mathcal{D}_2$$

where $\sigma > 0$ is a constant scalar and $\varphi_\xi(e)$ is a scalar function of the error. When the controller is in the *hit* state, the resulting shaped Hamiltonian assumes the following form:

$$\mathcal{H}^*(x,\omega) = \frac{1}{2}p^\top M^{-1}p + \frac{1}{2}\sigma\varphi_\xi(e)(q_1 - q_1^*)^2 + \gamma m_2 q_2$$

The main idea is to iteratively adjust the slope (steepness) of the energy of the system as function of the error. For instance, when $e > 0$, it can be derived that the robot had hit the ball with not enough momentum. Thus, at the next cycle, the energy function should be steeper so that the robot will accelerate faster toward the ball, since the dissipation rate k_d of the damping injection didn't change. The same concept can be adopted for $e < 0$, by making the the energy function less steep. A possible choice of $\varphi_\xi(e)$ is

$$\varphi_\xi(e) \triangleq \varphi_0 + a \cdot e(\xi) + b \cdot \sum_{i=1}^{\xi} e(i)$$

which provides a proportional and integral action with a constant offset φ_0 in response to the error. The integral action should ensure zero steady state error.

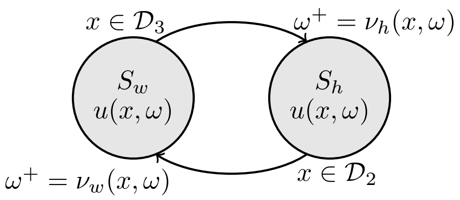

Fig. 2. Automata of the controller. It has an *hit* state $\mathcal{S}_h$ in which it forces the robot to move toward the ball and a *wait* state $\mathcal{S}_w$ in which it keeps the robot at a constant distance δ from the ball.The transitions happen as follows: $\mathcal{S}_h \to \mathcal{S}_w$ when the robot hits the ball, $\mathcal{S}_w \to \mathcal{S}_h$ when the ball reaches the peak of its bounce.

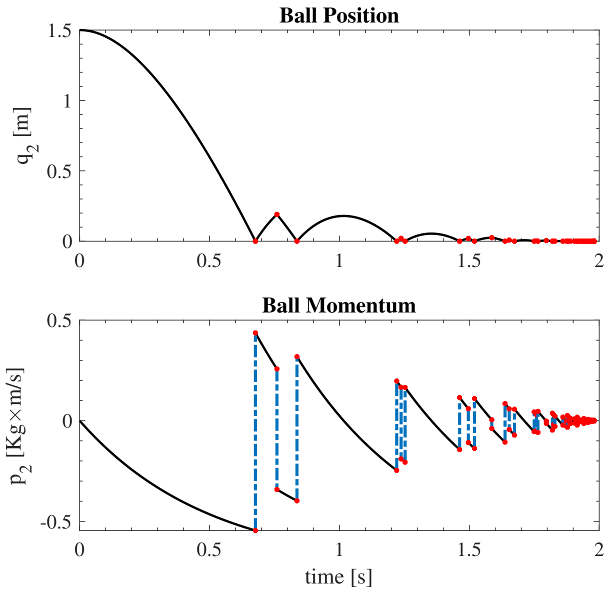

Fig. 3. Uncontrolled system: time evolution of the ball position momentum. Red dots correspond to system's jumps while dashed blue lines highlight discontinuous state changes. Notice that both the position and velocity goes asymptotically to zero.

5. NUMERICAL SIMULATIONS

To validate the proposed control scheme, numerical simulations have been performed. The simulations experiments have been carried out using Hybrid Equations (HyEQ) Toolbox Sanfelice et al. [2013] for the MATLAB environment. The physical parameters of the system have been chosen as $m_1 = 1\text{Kg}$, $m_2 = 0.15\text{Kg}$, $\beta_1 = 0.2\text{N}\cdot\text{s/m}$, $\beta_2 = 0.3\text{N}\cdot\text{s/m}$, $c_g = c_i = 0.8$. The initial conditions have been set to $q_1(0) = 2\text{m}$, $q_2(0) = 1.5\text{m}$, $p_1(0) = p_2(0) = 0\text{Kg}\cdot\text{m/s}$. Firstly, the behavior of the uncontrolled system has been simulated. The time evolution of the state variables are shown in Figs. 3 and 4, respectively. The time evolution of the Hamiltonian function is shown in Fig. 5. It can be noticed that $\mathcal{H}$ presents a monotonically decreasing trend, in both flows (due to the viscous friction terms) and jumps (since the restitution coefficients are less than one). Moreover, from the figures it is easily to verify the asymptotic stability and attractivity of the origin ($x = \mathbb{0}_4$) and the *Zeno behavior* of the autonomous system (see Goebel et al. [2009]).

Simulations of the system controlled via iterative energy shaping have also been performed. The control parameters have been chosen as: $k_{p,w} = 10^4$, $k_{d,w} = 10^3$, $\delta = 0.5$, $\sigma = 10$, $k_{d,h} = 10^2$, $\varphi_\xi(e) = 3000 + 10e + 300\sum_{i=1}^{\xi} e(i)$, $q^*_{2,max} = 1$. The time evolution of the state variables are shown in Figs. 6, 7 and 8. Notice that the trajectory of the ball becomes periodic (asymptotically), reaching at each bounce the desired peak $q^*_{2,max}$, proving the effectiveness of the proposed control scheme. Furthermore, the time evolution of the energy function is shown in Figs. 9 and 10. As expected also the energy becomes periodic. In fact, the energy at the beginning and at the end of each cycle (bounce) become exactly the same. Moreover, as shown in Fig. 11 the error goes to zero with the number of cycles. After about 20 bounces it becomes practically zero. On the other hand, the function $\varphi_\xi(e)$ converges exponentially to a positive constant value. A

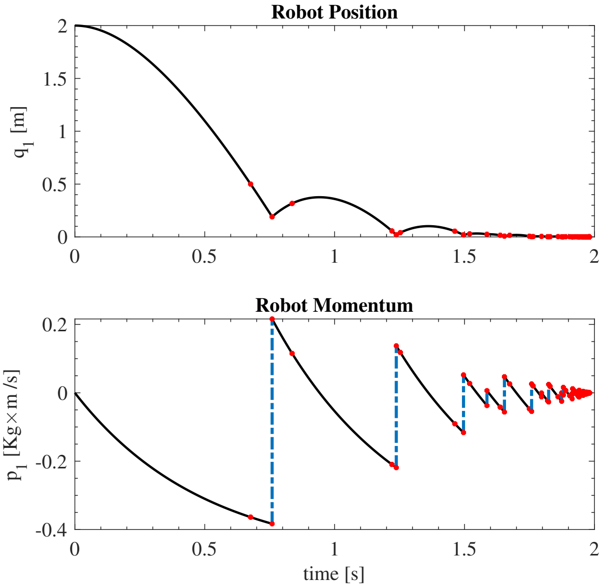

Fig. 4. Uncontrolled system: time evolution of the robot position momentum. Red dots correspond to system's jumps while dashed blue lines highlight discontinuous state changes. Notice that both the position and velocity goes asymptotically to zero.

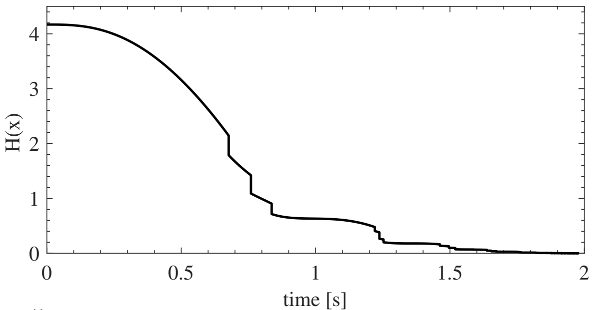

Fig. 5. Uncontrolled system: time evolution of the Hamiltonian function. Due to system's passivity, the energy monotonically decreases to zero both during flow and jumps.

final consideration about the convergence of the system to the desired periodic trajectory can be made from the phase-space portrait of the system's trajectory (Fig. 12). Being the time represented by the color transition, it can be noticed that the system approaches a limit cycle, an attractive asymptotically periodic trajectory, in which the tracking error is zero.

6. CONCLUSIONS

In this paper a new paradigm of energy based control for the ball–dribbling robot, the *iterative energy shaping*, has been introduced. Numerical simulations have been performed to prove the effectiveness of the proposed control scheme. Future work will include an additional formalization of the framework of *hybrid port–Hamiltonian* systems, e.g. well–posedness, Lyapunov thoery etc., and the exploration of the stabilization properties of the iterative energy shaping control for general hybrid systems.

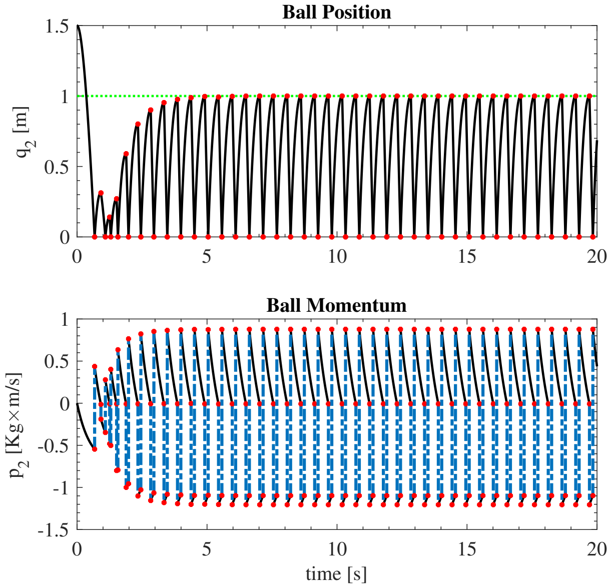

Fig. 6. Iterative energy shaping control: time evolution of the ball position and momentum. Red dots correspond to system's jumps while dashed blue lines highlight discontinuous state changes. Notice that the ball states converge on a periodic trajectory reaching at each bounce the desired peak $q^{*}_{2,max}$ * (dotted green line).

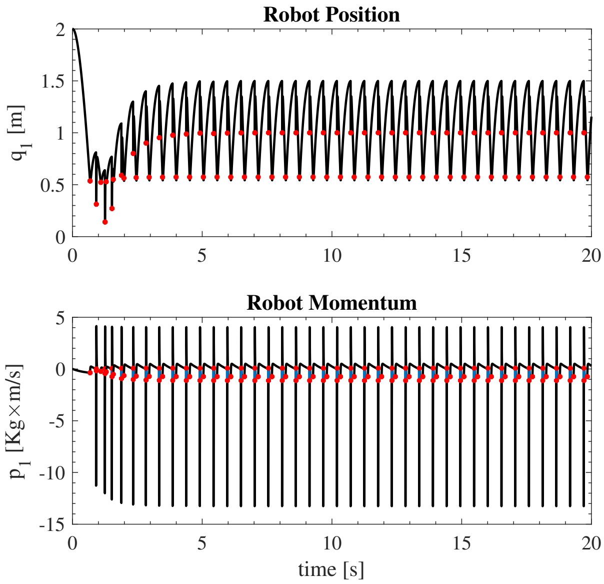

Fig. 7. Iterative energy shaping control: time evolution of the robot position and momentum. Red dots correspond to system's jumps while dashed blue lines highlight discontinuous state changes. Notice that both the position and velocity converge to a periodic orbit.

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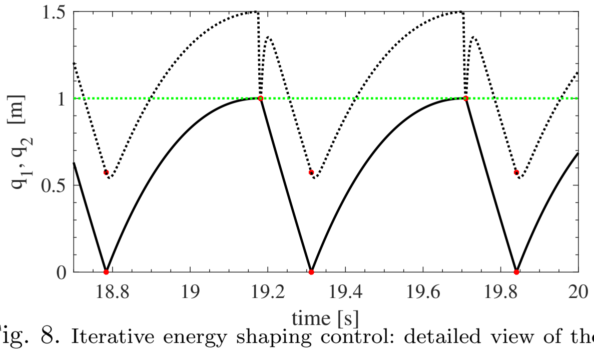

Fig. 8. Iterative energy shaping control: detailed view of the time evolution of the ball and robot positions at steady state: q_2 (solid black line) q_1 (dotted black line) and desired peak $q^*_{2,max}$ (dotted green line).

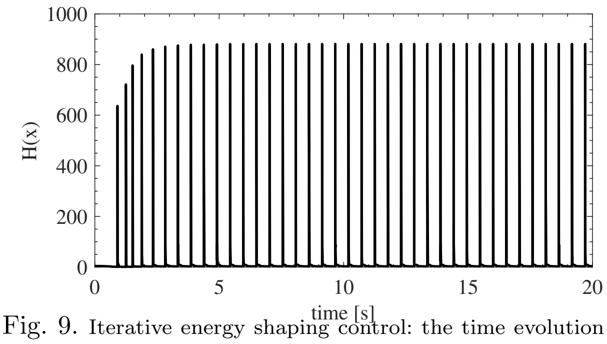

Fig. 9. Iterative energy shaping control: the time evolution of the Hamiltonian function. After a short transient (less than 5s) also the system's energy becomes periodic.

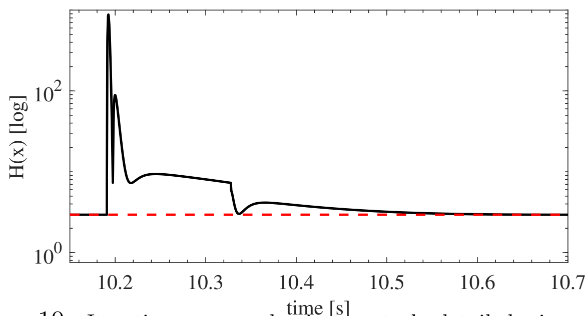

Fig. 10. Iterative energy shaping control: detailed view of the time evolution of the Hamiltonian function at steady state to highlight its periodicity.

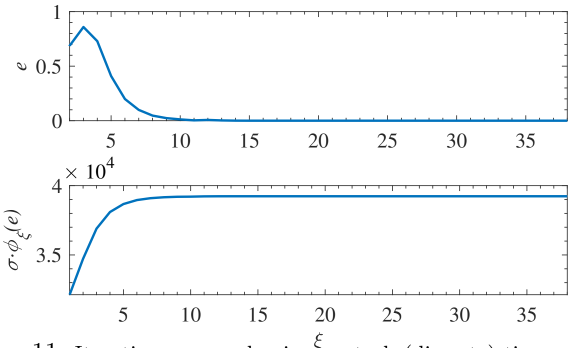

Fig. 11. Iterative energy shaping control: (discrete) time evolution of the tracking error e and of the energy shaping gain $\sigma\varphi_\xi(e)$. As the number of cycles ξ increases, the tracking error e, i.e. the difference between the desired and ball's bounce peak, goes asymptotically to zero. Instead, the energy shaping gain converges, increasing monotonically, to a constant value.

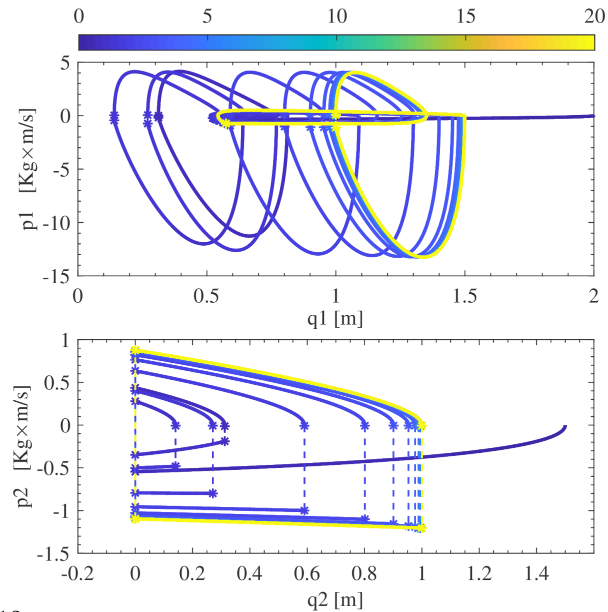

Fig. 12. Iterative energy shaping control: projections on the $q_1 - p_1$ plane (above) and the $q_2 - p_2$ plane (below) of the phase-space trajectory of the system (time is represented by color). The system converges to a hybrid limit cycle in which $e = 0$.

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