Robot Motion Planning Utilizing Local Propagation of Information Based on Particle Swarm and Its Internal Parameters

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Abstract—A new artificial potential field method for motion planning of mobile robot is developed in this paper. As a reactive motion of robot, a feedback oriented to the contour of repulsive potential function is applied with the ordinal attractive and repulsive component. Furthermore, inspired by Particle Swarm Optimization, particles as simplified virtual robots are utilized for motion planning. Each particle searches the space and transmit information regarding local stable region to the swarm, namely, the robot and other particles. Internal parameters of the robot and particles are also introduced to adjust the propagation of information locally gained. Simulation results demonstrate the robustness of proposed method against the complexity of an environment.

I. INTRODUCTION

Online robot motion planning is an important research issue for autonomous robots operated in an unknown or unpredictable environment. In real time autonomous navigation problem, robot must be capable of avoiding obstacles and reaching destination from initial state in a finite time interval. Enormous research has been developed and present robot navigation methods would be classified into two categories (or their combination). That is, global path planning and local reactive motion generation [4].

In global methods, environment is assumed to be completely known and the path is optimized in accordance with the whole map information [10] [11]. Derived path can lead the robot to the destination in a refined way. Furthermore, the path is collision and deadlock free, but they often require enormous computational cost and do not consider dynamics of robot. Therefore it might be ill-suited to adopt global method and recompute optimized path for each variation of a dynamic environment.

On the other hand, a whole environmental information does not utilized in local methods [2] [8]. Although the motion of robot is not necessarily optimal, local methods can be recomputed fast. In time varying and unpredictable environment, a capability to modify motion strategy rapidly is one of the most fundamental specifications. The motion of robot should be determined by the trade-off between destination oriented stretegy and collision avoidance oriented strategy. So a whole path efficiency depends the balance of strategies and therefore it is desirable for robot system to be able to recompute the motion at least in a comparable time interval with environmental change.

The authors are with the Department of Precision Engineering, Faculty of Engineering, The University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo, Japan. {masuyama, yamashita, asama}@robot.t.u-tokyo.ac.jp However, local methods has potential problem to get stuck in trap situations like U-shape obstacles. It is difficult to guarantee the global asymptotic stability in principle because the motion is computed by limited environmental information in limited time/spatial terrain. In this paper, we approach this problem through search actions of simplified virtual agents. Internal parameter of each agents entail social signal and the motion of robot is decided by sensory input and the spatially propagated information.

In the sequel, related works are discussed in Section II, and proposed method is presented in Section III. Simulation results are shown in Section IV. Finally, discussion and conclusion are summarized in Section V.

II. RELATED WORKS

Proposed method is constructed based on artificial potential field method. And ideas conceived by Particle Swarm Optimization is utilized to deal with some problems of artificial potential field method. In this section, artificial potential field method and Particle Swarm Optimization are briefly reviewed with the objective of robot motion planning.

A. Artificial Potential Field Method

Artificial Potential Field method is one of the most popular techniques for robot motion planning [2]. The fact is remarkable that this method can generate a smooth trajectory and be easy to analyze mathematically. Additionally, precise model of an environment is not necessarily needed. Although artificial potential field method has these accessible properties, there is local minimum problem of potential functions. A number of researches have been done to tackle with this problem, for recent example, [7] [9].

Let $q \in \mathbb{R}^n$ be an *n*-dimensional state of robot and $U_a : \mathbb{R}^n \to \mathbb{R}_+$ and $U_r : \mathbb{R}^n \to \mathbb{R}_+$ be attractive and repulsive potential function respectively. Then the motion of robot is planned by following equation.

$$\dot{\boldsymbol{q}} = -\nabla U_a(\boldsymbol{q}) - \nabla U_r(\boldsymbol{q}) \tag{1}$$

If we choose attractive potential function as Lyapunov function candidate, its time derivative is

$$\frac{d}{dt}U_a(\boldsymbol{q}) = -\|\nabla U_a\|^2 - \nabla U_a^T \nabla U_r \tag{2}$$

Thus (2) shows local equilibria can exists at such points $q_{le} \in \mathbb{R}^n$ that satisfies $-\|\nabla U_a(q_{le})\|^2 = \nabla U_a(q_{le})^T \nabla U_r(q_{le})$. So local equilibria are always contained in the subspace of \mathbb{R}^n , $\Omega = \{q : \nabla U_a^T \nabla U_r \leq 0\}$, because $-\|\nabla U_a\|^2$ is always less or equal 0. Therefore two strategies can be effective to prevent trapping in local minima. One is evasion of approach

to the terrain Ω and the other is erasing local stable region like downward convex region.

B. Particle Swarm Optimization

Particle Swarm Optimization (PSO) [1] is one of a metaheuristic optimization methods actively utilized for robot motion planning in recent years [3] [5] [6]. PSO is inspired by simplified social model of bird flocking and fish schooling. A set of randomly generated search points called particles construct swarm and it propagates through the space. Each particle adjusts its motion in accordance with the action history and information of search space shared across the particles. Although PSO is mathematically simple algorithm and does not need information regarding gradient of objective function, it can solve nonlinear optimization problem fast.

In this paper it is worthy of remark that the basic principle of original PSO is the hypothesis, "information is shared across the swarm". Each particles utilize information acquired not only by itself, but also other particles' to determine the search action. As a member of aggregation, it is reasonable to integrate the action pattern lead by own experience and the common sense for the purpose of adaptation to the environment.

III. ARTIFICIAL POTENTIAL FIELD METHOD UTILIZING PARTICLE SWARM

The robot is required to reach the goal state while avoiding collision with obstacles by real-time computation in a not fully-known environment. Particularly a local reactive method, artificial potential field method has suitable characteristics for this objective, but there is a local minimum problem. Two strategies to prevent the trapped situation in local stable region would be possible as described in Section II-A. In this section, proposed method is presented along the above strategies.

A. Artificial Potentilal Field Method with Contour Feedback

Here, we discuss the evasion of terrain that possibly incorporates the local stable equilibria. In artificial potential field, local minimum could exist in subspace $\Omega = \{q :$ $\nabla U_a^T \nabla U_r \leq 0$. $\nabla U_a^T \nabla U_r \leq 0$ means the gradient of attractive potential function and the gradient of summation of repulsive potential functions derived from each obstacle make an angle more than π . So roughly speaking, if $\nabla U_a^T \nabla U_r \leq 0$ is satisfied, obstacles are frequently found in the interspace between the robot and its destination. If rectilinear path is blocked by the obstacles, the robot must generate circumvent motion in accordance with negative gradient of U_r . The success and failure of circumvent motion depends on the existence of local minimum equilibria. Therefore it is desirable for the robot to set up not only repulsive operator but also a explicit module to circumvent the obstacles that operates regardless of existence of local minimum equilibria.

Whereat we introduce a feedback oriented to the contour of repulsive potential function. Let $J \in \mathbb{R}^{n \times n}$ be a skewsymmetric matrix, then the feedback \boldsymbol{u} is represented as

$$\boldsymbol{u} = \kappa \left(1 - \frac{\nabla U_a^T \nabla U_r}{\|\nabla U_a\| \|\nabla U_r\|} \right) \boldsymbol{J} \nabla U_r$$
(3)

 $\kappa \in \mathbb{R}$ is a parameter determines magnitude and direction of the circumvent motion. From (3), \boldsymbol{u} could be regarded as nonlinear damper and it takes zero if ∇U_a and ∇U_r take same direction. Hence \boldsymbol{u} can generate motions that evade the terrain Ω . Besides, it makes no effect for the risk of collision, because it takes direction along the contour of repulsive potential fuction. With the feedback, whole dynamics of planner is modified to $\dot{\boldsymbol{q}} = -\nabla U_a - \nabla Ur + \boldsymbol{u}$. Again, time derivative of U_a is

$$\frac{d}{dt}U_{a}(\boldsymbol{q}) = -\|\nabla U_{a}\|^{2} - \nabla U_{a}^{T}\nabla U_{r} + \kappa \left(1 - \frac{\nabla U_{a}^{T}\nabla U_{r}}{\|\nabla U_{a}\|\|\nabla U_{r}\|}\right)\nabla U_{a}^{T}\boldsymbol{J}\nabla U_{r}$$
(4)

 $\kappa \nabla U_a^T J \nabla U_r$ can be positive if direction of circumvent motion generated by u does not point a direction of the destination. However, (4) shows it must be required to recede from destination in some trapped situations to move to global stable terrain. In the result it turns out sometimes the robot should tolerate detriment in short time interval for the accession to the global minimum.

Introduced feedback works along the contour of $U_r(q)$. It could be interpreted as a circumvent motion or search motion to discover terrain $\Omega_c = \{q : \nabla U_a^T \nabla U_r > 0\}$. In Fig. 1(a), example of trajectory generated by the method is shown. Square is a robot, cross is obstacle, five-pointed star is destination and contour of $U_a + U_r$ is also depicted. The robot go straight and encounter obstacles, then it starts circumventing to search other candidate of global stable terrain. Finally the robot succeeded to reach the destination. Fig. 1(b) shows the robot could be trapped by local minimum in enclosed situation by the obstacles. This is reasonable function because the robot should not muscle in to weave its way through obstacles in such situations, especially when obstacle represents human.

B. Particle Swarm as Virtual Agents

The robot sometimes generates an inefficient motion in above method. As is shown in Fig. 1(a), if a wall stands between the robot and the destination, the robot would hustle against the repulsive potential field derived from the wall and finally start to circumvent it. Obviously the derived path is not the shortest one. This kind of problem is intrinsic for local reactive methods, because it makes decision without calculating the entire optimality. However, as mentioned in Section I, global computation is not realistic in terms of calculation cost. Therefore a method enables the robot to obtain helpful local information efficiently is needed.

Information of the past best postion in the swarm is shared across the particles in PSO. At the same time, the particles just conform simple dyanamics but does not compute global optimality. Nevertheless PSO can be a powerful solver for



Fig. 1. Artificial potential field method with contour feedback

an optimization problem. By metaphorical thinking of PSO, if the particles have dynamics as a virtual robot agent, it would be able to get information concerning the terrain that has the possibility to put the robot in undesirable situations. And if the particles can transmit the information of potential risk to the real robot in an appropriate way, the robot can utilize spatial information to decide its motion without global computation. PSO has its basis on best action search like foraging of living objects. In contrast, our idea has a basis on worst action search like crisis prevention.

Let the dynamics of particles be determined by usual attractive and repulsive potential function as (1). Then the particles would move toward the local or global minimum in accordance with their initial states. Therefore usage of just attractive and repulsive potential function provides spatially lopsided information regarding equilibria in stable terrain. To utilize spacious and various information, we introduce repulsive potential function $U_p^j : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+$. *j* is an index of *m*-tuple particles and $q_p^j \in \mathbb{R}^n$ is a state of *j*th particle. Then $U_p^j(q_p^i, q_p^j)$ is a repulsive potential function assigned to *j*th particle and acts on *i*th particle (when *q* is



Fig. 2. Contour of potential function compensated by three particles

chosen, $U_p^j(\boldsymbol{q}, \boldsymbol{q}_p^j)$ acts on the robot). A whole dynamics of *i*th particle is represented as

$$\dot{\boldsymbol{q}}_{p}^{i} = -\nabla U_{a}(\boldsymbol{q}_{p}^{i}) - \nabla U_{r}(\boldsymbol{q}_{p}^{i}) - \sum_{j \neq i}^{m} \nabla U_{p}^{j}(\boldsymbol{q}_{p}^{i}, \boldsymbol{q}_{p}^{j})$$
(5)

So each particle is attracted by global minimum and repelled by obstacles and other particles. Thus information of the search space could be collected in reasonable manner, because the particle moves as simplified real robot and repell each other to extend the search space. Another merit to use particles is that we have no need to consider the risk of collision, because they don't have any physical body. So the particles can search the space in a way real robot can not do.

Next we design the propagation of information regarding local stable region to the real robot. This could be done by the every assigned particles' repulsive potential function that works on the robot. Particles search the space in accordance with the dynamics (5) and might be trapped in local stable region. This means unreasonable paths in terms of ordinal artificial potential field method exist in neighborhood of the particle. So the particle should offer the information to the robot and other particles. That is, the particle generates potential field that repells robot and other particles so as not to approach the terrain. Fig. 2 depicts particles, represented by upward-pointing triangles, move to the local stable region and compensate it by assigned repulsive potential function.

With the effect of particles' repulsive potential function, dynamics of robot is modified as below

$$\dot{\boldsymbol{q}} = -\nabla U_a(\boldsymbol{q}) - \nabla U_r(\boldsymbol{q}) - \sum_j^m \nabla U_p^j(\boldsymbol{q}, \boldsymbol{q}_j) + \bar{\boldsymbol{u}}(\boldsymbol{q})$$
 (6)

$$\bar{\boldsymbol{u}} = \kappa \left(1 - \frac{\nabla U_a^T \nabla U_r}{\|\nabla U_a\| \|\nabla U_r\|} \right) \boldsymbol{J} (\nabla U_r + \sum_j^m \nabla U_p^j) \quad (7)$$

Hereat, u is replaced by $\bar{u} \in \mathbb{R}^n$ represented as (7) that also circumvent repulsive potential functions assigned to

the particles. The reason of modification is that U_p^j can erase local minimum derived from U_r , but they also could make new local stable regions usually smaller ones than the originally existed. Note that the robot is not assigned any repulsive potential function act on the particles.

C. Internal Parameters and Proposed Method

Framework of proposed method has been constructed at this point, but there could be some improvement from another point of view. As mentioned in Section III-B, proposed method has its basis on starategy of evading from worst situation. Hence it might be possible to consider generated motion reflects events that operate the "emotions" of the robot and particles, so as to disincline the situation within a context of analogy with living objects. Here we label the internal state as stress and discuss constructive utilization of such function to make our method more adaptive to the stressful situations.

Firstly, we focus on dynamics of internal mechanism inside the particles. The task assigned to the particles is to inform the robot and other particles a prospect about local stable region. The information is transmitted by the repulsive potential function U_p^i centered on the position of *i*th particle. Whereat, width of U_p^i should be conditioned by how undesirable the situation ith particle is in. If there are no obstacles around the particle, then U_p^i should have narrow skirts so as not to block a path of robot and other particles. That is, ith particle does not feel risk or stress and there are no information to share across the swarm. In contrast, if there are obstacles that block the path of *i*th particle, it would be stressed by the obstacles. So the information of stressful terrain should be informed across the swarm by wide skirted repulsive potential function. Here we set $b_p^i \in \mathbb{R}$ as a parameter of width of the U_p^i . b_p^i is adjusted by following equation.

$$b_p^i = \beta_p^i \tanh\left(\int_{t-T_p}^t \exp(-\lambda_p^i \|\dot{\boldsymbol{q}}_p^i\|) dt\right) \tag{8}$$

 $\beta_p^i \in \mathbb{R}$ and $\lambda_p^i \in \mathbb{R}_+$ are paremeters represent the maximum value of b_p^i and speeds of stress accumulation. $T_p \in \mathbb{R}_+$ is a time interval stress is holded. So the stress accumulated before $t - T_p$ is got lost in oblivion. (8) means *i*th particle broaden assigned repulsive potential function with it takes small velocity. In local stable region a particle takes small velocity and takes zero at equilibrium, so the formulation is constructed as U_p^i covers local stable region. Of course, U_p^i should not be expanded in neighborhood of global stable equilibrium and it is treated special case. Actually, in Fig. 2, stress parameters have been already intorduced and the transition of b_p^i is depicted in Fig. 3.

Secondly, we discuss the design of internal mechanism for real robot. Although the particles try to search the space and clarify risks of local stable region to the robot, there may be situations in that the robot is trapped by local minimum of compensated potential function $U_a + U_r + \sum_j^m U_p^j$. Alternatively, the robot might take cycled path and could



Fig. 3. Internal parameters of particles

not approach the destination. Both of the cases degenerates performance of the system, but they could occur in unpredictable environment. To overcome such stalled situations, the robot should be capable of interpreting the situations and breaking out from the terrains. And so we implement a stress parameter $s \in \mathbb{R}_+$ and function to produce new particles to vary the neighborhood environment of the robot.

$$s = \int_{t-T}^{t} \exp(\lambda \dot{\boldsymbol{q}}^T \frac{\nabla U_a}{\|\nabla U_a\|}) dt \tag{9}$$

 $\lambda \in \mathbb{R}_+$ is a parameter regarding accumulation of stress and $T \in \mathbb{R}_+$ is time interval during that the past stress is holded. If the robot could not course steepest decendent direction of U_a during time interval T and s exceeds the threshold $s_{th} \in \mathbb{R}_+$, then the robot lets out new particle, so the number of particles is updated to m+1. The value of s is initialized at the same time as new particle is produced. The sequence is repeated until the particles release the robot from stressful terrain.

Now then proposed method can be taken together by the equations (5), (6), (7), (8), (9). First of all, the robot place some particles in the space and they move toward the destination along an attractive potential function. The particles expand skirts of assigned repulsive potential function if they are blocked or stressed by the obstacles. The robot would produce new particle if it could not proceed to the destination continuously. In so doing, the motion of robot is generated by the balance of attractive and repulsive and circumventing components.

Note that there could be various design of stress functions for the robot and particles depending on the purpose of recusal. So (8) and (9) are only instances to exemplify the effectiveness of proposed method.

IV. SIMULATION RESULTS

In this section, results of two computational experiments are shown. One is an environment in that U-shaped obstacle is set. And the other is an environment in that multiple obstacles are set at random positions. Here, we utilize a following potential function. The destination is set at origin.

$$U_k(\boldsymbol{q}) = a_k \boldsymbol{q}^T \boldsymbol{q} \exp\left(-\frac{(\boldsymbol{q} - \boldsymbol{q}_k)^T (\boldsymbol{q} - \boldsymbol{q}_k)}{b_k^2}\right)$$
(10)

 $a_k \in \mathbb{R}_+$ and $b_k \in \mathbb{R}$ are parameters. When we take $q_0 = \mathbf{0}$ and take b_0 large enough, U_0 is treated as $U_a := U_0$. And (10) can be repulsive potential function if we take q_k as

$$m{q}_k = rac{\|m{q}_o^i\|^2 - b_i^2}{\|m{q}_o^i\|^2}m{q}_o^i$$

 $q_o^i \in \mathbb{R}^n$ is position of *i*th obstacle and then U_k has its local maximum at q_o^i . U_p^j is represented in the same way for q_p^j . Initial position of placed particle is determined as described below.

$$oldsymbol{q} + r_d rac{oldsymbol{q}_d - oldsymbol{q}}{\|oldsymbol{q}_d - oldsymbol{q}\|} + r_u oldsymbol{f}_u$$

 $q_d \in \mathbb{R}^n$ is destination and $f_u \in \mathbb{R}^n$ is a function outputs vector that has uniformly distributed elements in the interval (-1, 1). $r_d \in \mathbb{R}_+$ and $r_u \in \mathbb{R}_+$ are positive parameters. Particles basically precede the robot, because they should not affect the robot motion immoderately and their function is searching of potential field. In parallel, particles need to help the robot when it gets stuck in local stable region, so it would be desirable to set the initial position of particle, or r_d and r_u , at front of robot, not so far position.

A. U-shaped obstacles

To show the characteristic features of proposed method, an environment set U-shaped obstacle is simulated. Following set of initial parameters are chosen. $a_0 = 0.5$, $b_0 = 400$, initial state of robot $\mathbf{q}_0 = (0,0)$, $a_p^j = 0.5$, $b_p^j = 0.001$, $r_d = 3$, $r_u = 1.5$, m = 4. Positions of obstacles are (6,6), (5,7), (7,5), (4,8), (8,4), (3,7), (7,3), (2,6), (6,2), (1,5), (5,1), and corresponding (a_o^i, b_o^i) are (1.5, 1), (1.4, 1), (1.4, 1), (1.2, 1), (1.2, 1), (0.9, 1), (0.9, 1), (0.8, 1), (0.8, 1), (0.4, 1), T = 2, $T_p = 2$, $\mathbf{J} = [0 \ 1; -1 \ 0]$, $s_{th} = 1.8$, $\lambda = 1$, $\beta_p^i = 1$, $\lambda_p^i = 0.1$, $\kappa = 0.5$. Additional particles are set by $r_d = 2$, $r_u = 1$. And the velocity of robot and particles have their maximum at 1 and 2 respectively. Sampling rate is 100[ms] and whole simulation time is 27[s].

Resulted trajectories of robot and particles are depicted in Fig. 4. Square represents robot, five-pointed star does the destination, crosses do obstacles, upward-pointing triangles do initial particles, and downward-pointing triangles do added particles respectively. Contour of $U_a + U_r + \sum_j^m U_p^j$ is also depicted. In Fig. 4(a), robot and initial particles started approaching to the destination and go into interior of Ushaped obstacle. Obviously there exists local stable region, so b_{p}^{i} and s increase gradually. Then the preceding particles compensate the local stable region by increase of b_p^i , and the robot starts circumvent motion without searching back of it. However, In Fig. 4(b), the robot moved few distances, because the local stable region could not be fully compensated just by the initial particles. So the robot is stressed and produce new particles in front at short time intervals. Finally in Fig. 4(c) the robot succeeded to circumvent Ushaped obstacle and arrived global stable terrain.

TABLE I Number of succeeded trial and success rate

	Original	Contour	Proposed
Case 1	210 (70.0%)	267 (89.0%)	275 (91.7%)
Case 2	67 (22.3%)	251 (83.7%)	279 (93.0%)

If we set more particles initially, generated path would be pre-circumvent motion and the performance should improve. Thus we can generate efficient motion of robot using appropriate setting of particles. And even if improper settings is done, the robot would produce particles in accordance with its stress parameter. This can be seen in Fig. 5 that depicts the transition of internal parameters through the experiment. From the top of the figure, s and b_p^i of two initial particles and b_p^i of three added particles are shown.

B. Randomly positioned obstacles

Obstacles are located in randomly determined positions in following experiments. In particular, the initial robot position and the destination are fixed at (0,0) and (10,10)respectively, and obstacles are uniformly distributed in an open interval (1,9) with respect to each axis. Number of obstacle is also chosen as uniform random numbers from 1 to 10 in Case 1 and 11 to 20 in Case 2. Number of trial is 300 and each trial is executed at a time interval [0, 30]. Other parameters are the same as the experiment of Section IV-A. If $\|\boldsymbol{q}_d - \boldsymbol{q}\| \leq 1$ is satisfied at t = 30, the trial is judged as success. Proposed method and original artificial potential field method and original method with contour feedback are tested. The number of succeeded trials and success rates are tabulated in Table I. It is remarkable proposed method have kept high success rate even though the complexity of the environment increased.

V. CONCLUSION

In this paper, artificial potential field method utilizing particle swarm and its internal parameter has been presented. Particles embedded dynamics of ordinal artificial potential field method search local stable region. Each particle accumulates stress as an internal parameter in local stable region and it broaden assigned repulsive potential function along with increase of the internal parameter. Additionally, particles are added in accordance with the internal parameter of the robot to vary the stressful environment. The robot plan its motion with the compensated artificial potential function and a feedback oriented to contour of sum of obstacles' and particles' repulsive potential functions. Computational experiments showed the effectiveness of proposed method especially in complex environment.

Narrow skirted and plentiful particles would generate precise motion, but there is a trade-off between computational cost and performance, so the evaluation of this term is future work. And parameter of contour feedback κ is fixed in this paper. κ controls the direction and weight of circumvent motion. Obvioulsy performance of system could improve if it is selected appropriately. As a expanded virtual robot



(b) From t = 5 to t = 10



Fig. 4. Generated motion for U-shaped obstacle

body, or a kind of inverse model, another type of particle would be available to determine κ . Finally, this method is originally invented as reference input for control system of mobile robot. We are going to construct comprehensive system supposed for dynamic environment in the future.



Fig. 5. Transition of internal parameters

VI. ACKNOWLEDGMENTS

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