# An Efficient Improved Artificial Potential Field Based Regression Search Method for Robot Path Planning

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***Abstract*** **- Path planning field for autonomous mobile robot is an optimization problem that involves computing a collision-free path between initial location and goal location. In this paper, we present an improved artificial potential field based regression search (Improved APF-based RS) method which can obtain a global sub-optimal/optimal path efficiently without local minima and oscillations in complete known environment information. We redefine potential functions to eliminate non-reachable and local minima problems, and utilize virtual local target for robot to escape oscillations. Due to the planned path by improved APF is not the shortest/approximate shortest trajectory, we develop a regression search (RS) method to optimize the planned path. The optimization path is calculated by connecting the sequential points which produced by improved APF. Amount of simulations demonstrate that the improved APF method very easily escape from local minima and oscillatory movements. Moreover, the simulation results confirm that our proposed path planning approach could always calculate a more global optimal/near-optimal, collision-free and safety path to its destination compare with general APF. That proves our improved APF-based RS method very feasibility and efficiency to solve path planning which is a NP-hard problem for autonomous mobile robot.**

***Index Terms – Autonomous mobile robot. Path planning. Artificial potential field. Regression search.***

## I. INTRODUCTION

In the last few decades, robotic scientists have investigated on service mobile robots which could be able to operate within human-robot coexistent environments to execute different complex works, such as transportation of heavy objects, surveillance, rescue, and guiding people in exhibitions and museums. Autonomous mobile robot path planning or navigation is one of the most important applications for robot control systems and has attracted remarkable attention from number of researchers. Path planning is aimed at enabling robots with the capabilities of automatically deciding and executing a sequence of collision-free and safety motions in order to achieve a certain tasks in a given environment. As described in many interesting researches, two importance features that distinguish these algorithms are whether the environment is known or unknown and whether it is static or dynamic.

Known environments are those in which all information about obstacles and targets are known a priori, the motion of robot is designed from the given information. Examples of successful path planning algorithms in this know environment include sub-goal network [1], cell decomposition [2], A* [3] and D* algorithm [4], traditional artificial potential field [5], and many others [6-8]. In unknown environments, robot does not have any previous knowledge about environment or only partial information is available. Therefore, robot must plan a path based on a few available information or local sensing information. In recent years, lots of researchers have achieved important investigation results in such critical environment, for instance, genetic algorithm [9, 10], simulated annealing [11], ant colony optimization algorithm [12], and others [13, 14]. This paper presents a new efficient approach for autonomous mobile robot path planning in complete known environments. Herein, we propose an improved artificial potential field based regression search method (improved APF-based RS method) for mobile robot path planning which can program a shorten solution from the location of robot to the position of target.

The remainder of this paper is structured as follows. The next section discusses related works for autonomous mobile robot path planning. Section III presents our improved APF to deal with local minima and oscillatory movements problems, and utilizes regression search method (RS method) to optimize the planned path. To demonstrate our method, amount of simulations are done in section IV. Finally, section V draws conclusions and sketches future work.

## II. THE RELATED WORKS

Large part of autonomous mobile robot path planning is pertaining to scheduling and routing, and is well-known to be NP-hard (NP-complete) problem. Path planning algorithms are classified as classic and heuristic approaches [15]. Classic algorithms aim to calculate an optimal solution if one exists, or prove that there is no feasible path. However, most of classic approaches are based on free configuration space (C-space) concept. In addition to their lack of adaptively and robustness, thus conventional approaches are not suitable for dynamic environments since utilizing a sequential search algorithm to generate a single solution which may become infeasible when environment changes, a new solution has to be generated from scratch. Expect for, the greater the dimension of free C-space, the more complex and computationally expensive the path planning problem will be. On the other hand, heuristic algorithms attempt to find search for a good quality solution in a short time. However heuristic algorithms may fail to find a good solution for difficulty problem, deadlock and oscillation happen easily.

The artificial potential field (APF) is firstly introduced by Khatib [5]. The potential function can be defined over free C-space as the sum of attractive potential pulls robot toward the goal configuration, and repulsive potential pushes robot away from obstacles. APF has often represented a good quality to achieve a fast and reactive response to dynamic environment. However, this method has been widely demonstrated that it suffer from unavoidable drawbacks which are very likely for robot to get trapped into a local minimum and oscillations. Paper [16] describes a hybrid approach, which integrates a priori knowledge of environment with local perceptions in order to execute the assigned tasks efficiently and safely. The results indicate that this method guarantees the robot can never be trapped in deadlocks even when operating within a partially unknown dynamic environment. In spite of its good properties, the navigation system described in this paper has typical drawback that is the system is relying on local perceptions and navigation strategies. Another improved APF is proposed in [17] utilizing quantum particle swarm optimization for rapid global searching and realizing the optimal path planning. They employ quantum particle swarm optimization to modify the parameters of APF for adapting different environment and dynamic obstacles. To address the local minima problem in the traditional APF, a method composed of robot regression and potential field filling is proposed [18-19]. The similar methods propose in [20-21], before calculating the resultant force that is put on an object in the potential field, they build links among closed obstacles to optimize the planed solution. Other kinds of improved artificial potential fields are investigated, such as in [22], they introduce the relative distance between robot and target into repulsive force function and modify the repulsion direction to ensure the global minimum is at the position of target. Paper [23] researches learning reactive and planning rules into mobile robot path planning. The main distribution of [24-25] is that apply virtual local target to guide robot escapes local minimum.

While all the mentioned above APF and its improved methods still suffer from many drawbacks, such as high time complexity in high dimensions that result in these methods could deal with real-time path planning, and some methods do not completely solve local minima and oscillation which makes them inefficient in practice. Moreover, the path under previously methods is not optimal/near-optimal, but only feasible for robot to adapt the given environment. In other words, robot move along the planed path will consume more energy and costs. As described in this paper, we present a efficient improved APF-based RS method which can obtain a global optimal/near-optimal path without local minima and oscillations in complete known environment information.

## III. THE PROPOSED PATH PLANNING METHOD

### A. Traditional Artificial Potential Field

The basic idea of APF method assumes that robot as a point moves in an abstract artificial force field. The artificial field in environment is composed of attractive field of target and repulsive field of obstacles. Attractive field is produce by target and direct to target point, while repulsive field is the synthesis repulsive field of different obstacles and the direction of the synthesis repulsive field is away from obstacles. Therefore, the potential function (1) is the APF of robot which is defined as the resultant of attractive field and repulsive field. Robot controls its movement toward the target point along the direction of APF. Under the method of APF, robot could find a collision-free path by searching the route along the decline direction of potential function.

The coordinate of robot is $q=(x, y)^T$, thus the APF is defined as:

$$U(q) = U_{att}(q) + U_{rep}(q) \tag{1}$$

Where: $U(q)$ is artificial potential field. $U_{att}(q)$ is attractive field. $U_{rep}(q)$ is repulsive field.

The negative gradient of APF is defined as artificial force which is the steepest descent direction for guiding robot to target point. Attractive force is the negative gradient of attractive field, and repulsive force is the negative gradient of repulsive field.

Thus, the artificial force of robot is:

$$F(q) = -\nabla U(q) = -\nabla U_{att}(q) - \nabla U_{rep}(q) = F_{att}(q) + F_{rep}(q) \tag{2}$$

Where: $F(q)$ is artificial force. $F_{att}(q)$ is attractive force. $F_{rep}(q)$ is repulsive force.

The attractive field between robot and target is constructed to pull robot to the goal area. The attractive field created by the goal is given by

$$U_{att}(q) = \frac{1}{2}k(q-q_g)^2 = \frac{1}{2}k\rho_{goal}^2(q) \tag{3}$$

Where: $k$ is a positive coefficient for APF. $q_g=(x_g, y_g)^T$ is the location vector of target. $\rho_{goal}(q) = \|q-q_g\|$ is the Euclidean distance from the location of robot to the position of target.

The attractive force on robot is calculated as the negative gradient of attractive potential field and takes the following form:

$$F_{att}(q) = -\nabla U_{att}(q) = -\frac{1}{2}k\nabla\rho_{goal}^2(q) = -k(q-q_g) \tag{4}$$

$F_{att}(q)$ is a vector directed toward $q_g$ with magnitude linearly related to the distance from $q$ to $q_g$. The components of $F_{att}(q)$ are the minus directional derivatives of the attractive potential along the $x$ and $y$ directions. Therefore, when the attractive potential takes effect, the components can be written as:

$$\begin{aligned} F_{att-x}(q) &= -k(x-x_g) \\ F_{att-y}(q) &= -k(y-y_g) \end{aligned} \tag{5}$$

Where: $F_{att-x}$ is the attractive force on the $x$ direction. $F_{att-y}$ is the attractive force on the y direction.

Robot should be repelled from obstacles, but when robot is far from obstacles, we do not want obstacles to affect robot's motion. Khatib uses the function (6) as the repulsive potential field.

$$U_{rep}(q) = \begin{cases} 0 & ,\rho(q) \geq \rho_0 \\ \frac{1}{2}\eta(\frac{1}{\rho(q)} - \frac{1}{\rho_0})^2 & ,\rho(q) \leq \rho_0 \end{cases} \tag{6}$$

Where: $\eta$ is a positive scaling factor. Let $q_c=(x_c, y_c)$ be unique configuration in obstacle closest to $q$. $\rho(q) = \|q-q_c\|$

is the shortest distance between robot and obstacle. $\rho_0$ is the largest impact distance of single obstacle. There is no impact for robot when the distance between robot and obstacle is greater than $\rho_0$. Similarly to the attractive force, the repulsive force is the negative gradient of repulsive potential function, as follows:

$$F_{rep}(q) = -\nabla U_{rep}(q) = \begin{cases} 0 & ,\rho(q) \geq \rho_0 \\ \eta(\frac{1}{\rho(q)} - \frac{1}{\rho_0})(\frac{1}{\rho^2(q)})\nabla\rho(q) & ,\rho(q) \leq \rho_0 \end{cases} \tag{7}$$

$$F_{rep}(q) = \begin{cases} 0 & ,\rho(q) \geq \rho_0 \\ \eta(\frac{1}{\rho(q)} - \frac{1}{\rho_0})(\frac{1}{\rho^2(q)})\frac{q-q_c}{\|q-q_c\|} & ,\rho(q) \leq \rho_0 \end{cases} \tag{8}$$

$F_{rep\text{-}x}$ and $F_{rep\text{-}y}$ are the Cartesian components of the repulsive force $F_{rep}$. When the repulsive potential acting on robot takes effect, the components can be written as:

$$F_{rep-x}(q) = \begin{cases} 0 & ,\rho(q) \geq \rho_0 \\ \eta(\frac{1}{\rho(q)} - \frac{1}{\rho_0})(\frac{1}{\rho^2(q)})\frac{x-x_c}{\|q-q_c\|} & ,\rho(q) \leq \rho_0 \end{cases} \tag{9}$$

$$F_{rep-y}(q) = \begin{cases} 0 & ,\rho(q) \geq \rho_0 \\ \eta(\frac{1}{\rho(q)} - \frac{1}{\rho_0})(\frac{1}{\rho^2(q)})\frac{y-y_c}{\|q-q_c\|} & ,\rho(q) \leq \rho_0 \end{cases} \tag{10}$$

While there are many obstacles in the environment, the total repulsive potential field is the sum of all obstacles' repulsive potential field. The total potential field can be expressed as function (11).

$$U(q) = U_{att}(q) + \sum_{i=1}^{n} U_{rep}(q) \tag{11}$$

Where: $n$ is the number of obstacles.

The total artificial force is:

$$F(q) = F_{att}(q) + \sum_{i=1}^{n} F_{rep}(q) \tag{12}$$

Although the traditional APF method can plan smooth path effectively, it has fatal problems. When the attractive force and repulsive force is equal or almost equal and collinear but on the opposite direction in the process of moving to target, the potential force of robot is zero, then it will cause robot to be trapped in local minima and oscillations ( Fig.1 (a) and (c)). And when the position of target is very close to obstacles, robot could not reach the target (Fig.1 (b)).

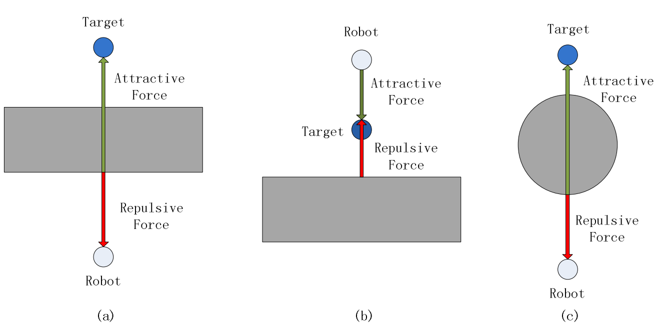


Fig.1 Local minima

### *B. Improved Artificial Potential Field*

The value of attractive field between robot and target is decided by their distance $\rho_{goal}(q)$, as function (3) presented. While when $\rho_{goal}(q)$ is very great, the attractive force will become very great too. In other words, when robot is far away target, it is easily leading robot move too close toward the obstacles [26]. Therefore, in the real environment shown in Fig.2, robot has the risk of collision to obstacles when we take account the error of path planning. Thus, the attractive field and attractive force are modified as function (13) and (14).

$$U_{att}(q) = \begin{cases} \frac{1}{2}k\rho_{goal}^2(q) & ,\rho_{goal}(q) \leq d \\ dk\rho_{goal}(q) & ,\rho_{goal}(q) \geq d \end{cases} \tag{13}$$

and

$$F_{att}(q) = \begin{cases} -k(q-q_g) & ,\|q-q_g\| \leq d \\ -dk\frac{(q-q_g)}{\|q-q_g\|} & ,\|q-q_g\| \geq d \end{cases} \tag{14}$$

Where, $d$ is positive coefficient for attractive field and force.

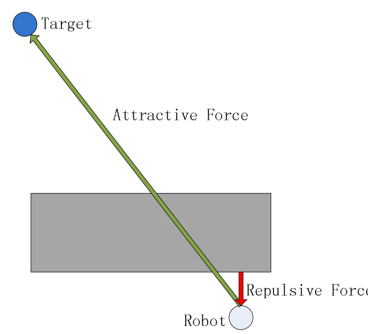


Fig.2 Attractive potential field

For the local minima of Fig.1 (a), we employ virtual local target to guide robot escapes from local minima, and this method can resolve oscillations. The detail method of virtual local target is presented in [24]. In this paper, we utilize function (15) and (16) to deal with robot non-reachable target (Fig.1 (b)), we name it as repulsive force disappearance method. Once robot detects the distance between target and obstacle is less than $d_{Ob}$, and the distance between target and robot is less than $d_{gr}$, robot only move along the attractive field and attractive force instead of considering the resultant of attractive field/force and repulsive field/force.

$$U(q) = \begin{cases} U_{att}(q) + U_{rep}(q) & ,Otherwise \\ U_{att}(q) & ,\rho(q) \leq d_{Ob} \quad and \quad \rho_{goal} \leq d_{gr} \end{cases} \tag{15}$$

and

$$F(q) = \begin{cases} F_{att}(q) + F_{rep}(q) & ,Otherwise \\ F_{att}(q) & ,\|q_g - q_c\| \leq d_{Ob} \quad and \quad \|q-q_g\| \leq d_{gr} \end{cases} \tag{16}$$

All previously proposed APF and improved APF methods do not explicitly define the repulsive field about vertex of polygonal obstacles. As described by general APF, the direction of repulsive field of obstacles is the vertical direction of polygon and away from the obstacles as Fig.3 (a) shown, thus it will be unreasonable due to there is no repulsive field near the vertexes of polygonal obstacles [27]. Therefore, we define the repulsive field about vertex of polygonal obstacles like Fig.3 (b) and the direction is the tangential line of semicircle [28]. Similarly, we change the direction of repulsive field which is caused by circle obstacles (Fig.4) to solve the problems of general APF, local minima and oscillatory movements.

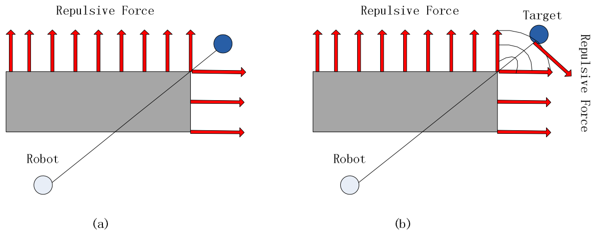


Fig.3 Repulsive field of polygonal obstacle

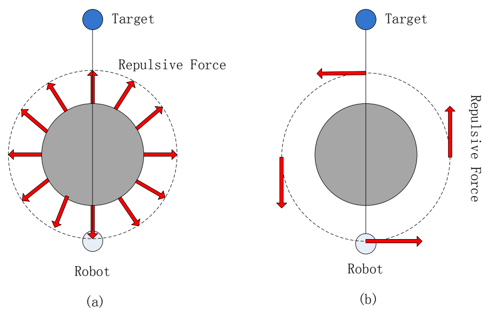


Fig.4 Repulsive field of circle obstacle

### C. *Improved APF-based Regression Search Method*

Although our improved APF which could resolve the local minima and oscillations successfully, but the key problem is that apply all APF methods including our method could not plan an optimal/near-optimal path both in complete/partial known environments. This shortcoming makes the applications of such method are very limited, especially for the time/energy constrain robot. While another contribution of this paper is that we develop an improved APF-based RS method to optimize the planned path. The optimization path is calculated by connecting the sequential points which produced by APF.

From the location of robot to destination, the inter-start point connects with the latter inter-point as a straight line sequentially. If this line does not across any obstacles, then the inter-start point re-connects with the next latter point as a new straight line until this line across an obstacle or the distance between this line and obstacles is less than $D_0$. Saving this connected line as robot local optimal path from the inter-start point to the terminative point. After that system produces the next new straight line from the last terminative point as the next inter-start point to the latter point as the above mentioned does.

We use Fig.5 as an example to illustrate the regression search method based on our improved APF. Assumption that $T_i \in \{T_1, T_2, T_3 \cdots, T_i, T_{i+1}, \cdots, T_n\}$ are the sequential points which planned by artificial potential field, that is robot moves along the sequential points can reach the target point without colliding obstacles. Based on the regression search method, firstly, the initial point $T_1$ as inter-start point connects the next point $T_2$ as a straight line $L_{1,2}$. Then this method judges $L_{1,2}$ crossing any obstacles or not, or the shortest distance $D$ between $L_{1,2}$ and obstacle is or not less than $D_0$. If $L_{1,2}$ does not crossing any obstacles or $D$ is greater than $D_0$, then system re-connect $T_1$ with $T_3$ as $L_{1,3}$, and do the similar step mentioned above. Until $L_{1,i+1}$ because of $L_{1,i+1}$ crossing obstacle, so the feasible local optimal path is $L_{1,i}$, that means $T_i$ is the terminative point. Due to $T_{i+1}$ is not the last point, so the next inter-start point is $T_i$ and connects with the next point $T_{i+1}$ similarly. Therefore, the optimal path of this example is the line $L_{1,i}$ and $L_{i,n}$. In other words, robot move along $L_{1,i}$ and $L_{i,n}$ will consume the least energy according to the planned path, the distance of $L_{1,i}$ and $L_{i,n}$ is the shortest.

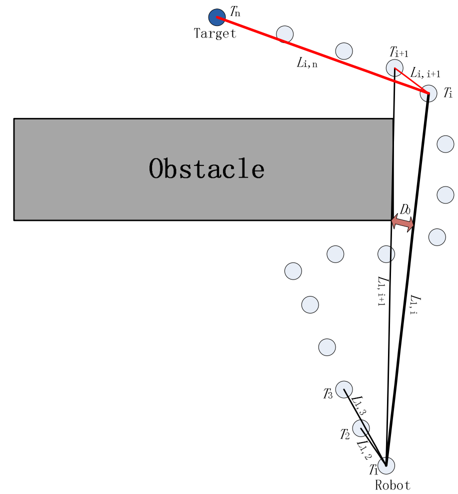


Fig.5 Regression Search Method

The entire algorithm of our proposed method is as follow.

1. *Compute the artificial potential force F(q) at current configuration under our proposed improved artificial potential field.*
2. *Take a small step in the direction indicated by artificial potential force.*
3. *Save the coordinate as $T_i$.*
4. *Repeat until reach goal configuration.*
5. *Regression searches the sequential $T_i$.*
6. *From the inter-start point $T_i$, connects the latter point $T_j$ as a line $L_{i,j}$ sequentially.*
7. *If $L_{i,j}$ is a feasible line path, then re-connects $T_i$ and $T_{j+1}$.*
8. *Else save $L_{i,j}$, then the point $T_j$ as the next inter-start point and connects the latter point sequentially.*
9. *Return to step 6 until the final point.*
10. *Obtain the optimal path.*
11. *Robot moves along the optimization path.*

## IV. EXPERIMENTS AND RESULTS

### A. *Simulation Environment Settling*

Some simulation experiments are carried out for validing the proposed algorithm using VC++. The environment is setting as square with width of 20 *m*, a free configuration space (free C-space) which shown in Fig.6. The coefficient $k$ for calculating attractive field is 0.3. To prevent the planed path is far away obstacles enough, we set the positive coefficient $d$ is 3. The positive scaling factor of repulsive field $\eta$ is 2.0. The largest impact distance for mobile robot from obstacles $\rho_0$ is 0.5. The distance $d_{Ob}$ between obstacles and target is 0.4 and $d_{gr}$ is 0.6, which is settling to solve the target non-reachable problem. For obtaining an optimal collision-free path based on the improved artificial potential field, the $D_0$=0.2 is utilized. We assume that the moving step of robot is 0.1.

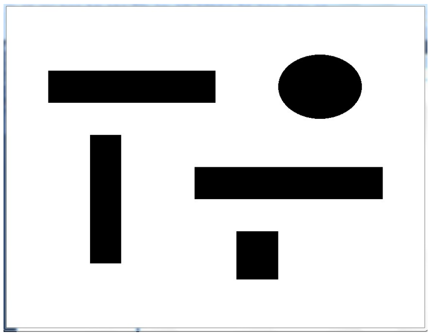

Fig.6 Simulation environment

### B. Improved Artificial Potential Field

This section describes the results obtained in various experiments performed under our proposed improved APF-based RS method to resolve the local minima and oscillatory movements. As Fig.7 shown, when the attractive field and the repulsive field in collinear and the directions are opposite (Fig.7 (a)), robot will fall into local minima using conventional methods, this is a kind of undesirable solution for autonomous mobile robot. However, the proposed method is very good at handling such local minimum problem by using virtual local target. Additionally, virtual local target can deal with oscillations well in Fig.7 (b), it also indicates that our method can plan a collision-free and safely path to target even the target is close toward obstacles. We mentioned above that conventional methods do not discuss the repulsive field about vertex of polygonal obstacles which is one of the normal reasons lead robot to local minima and oscillations. In this paper, we implement tangential line of semicircle and change the direction of repulsive field to eliminate it, as shown in Fig.7 (c) and Fig.7 (d).

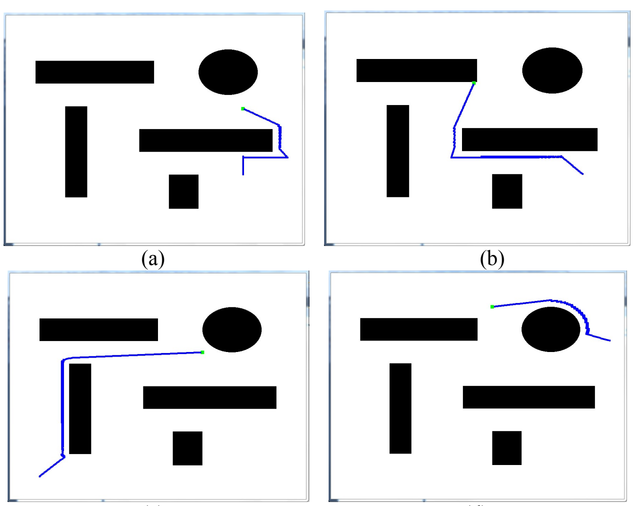


Fig.7 Solving local minima and oscillations

### C. Optimization of the planed path

As we know that the most important evaluation of path planning method for autonomous mobile robot is distance. To reduce the distance of planned path, many kinds of heuristic path planning methods are proposed, but the costs of these heuristic methods are the greatly time computation and complex structure. On the contrary, APF methods are efficient, less computational time and simplest mechanism, while the computed path of APF methods is not optimal/near-optimal which limits these methods to apply to time/energy constraint robot. In this paper, we proposed a regression search method to optimal the planed path, the results are shown in Fig.8, Fig.9 and Fig.10.

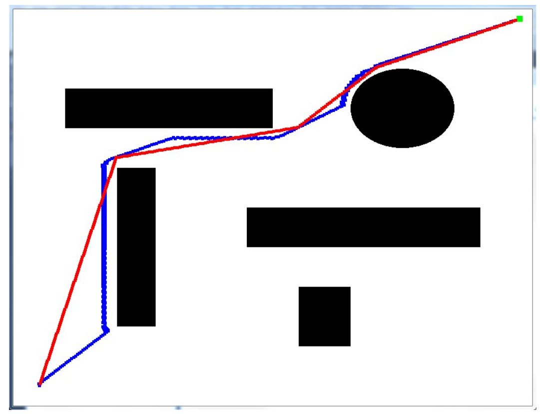

Fig.8 Planned path using our proposed method

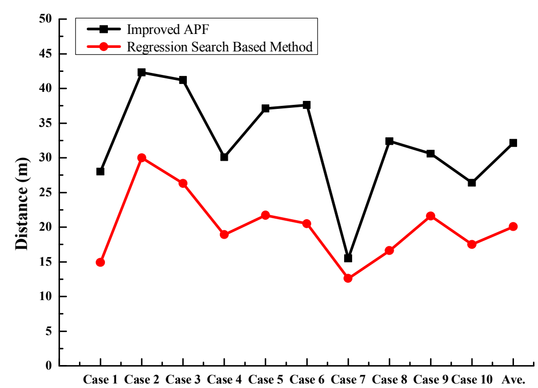


Fig.9 Distance of planned path

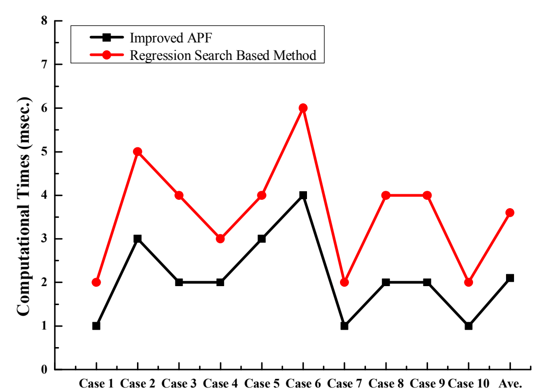


Fig.10 Computational times

In Fig.8, blue line is the path which is planned by our improved APF, while red line is the optimal path utilizing regression search method. Fig.9 shows the distance of planned path by only improved APF and improved APF-based RS method. From the figure we can see that our proposed algorithm greatly reduces the distance of planned path from the location of robot to the position of target, the results demonstration that the proposed method is very efficiency for general APF to optimize the planned path. While our method consumes a few more computational times (as Fig.10 shown) compare with conventional methods.

## V. Conclusion

Path planning problem is one of the most important robotic problems for autonomous mobile robot to accomplish given tasks. An improved artificial potential field based regression search method was proposed to obtain a global optimal/sub-optimal path without local minima and oscillations in complete known environment information. Virtual local target and repulsive force disappearance method are utilized to eliminate local minimum caused by traditional APF when attractive force and repulsive force in collinear but opposite direction. Oscillations problem is resolved by tangential line of circle when robot moves nearby the vertex of polygonal obstacles and circular obstacle. Due to the computed path by improved APF is not the shortest trajectory, we developed a regression search method to optimize the planned path, and proved that a safely, shorten and collision-free path for autonomous mobile path could be produced by amount of simulations. That proved our improved APF-based RS method is very feasibility and efficiency to solve path planning.

In the future works, we attend to improve the smoothness of the planning path, utilize the improved APF-based RS method for dynamic environment, moving target real-time path planning. We also believe our method can deal with unknown /partial information, local sensing path planning.

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